Preprocessing in displacement space for on-shore seismic processing: removing ground roll and ghosts without damaging the reflection data

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SUMMARY

This paper derives an elastic Green's theorem wave separation method for on-shore data in displacement space. Applying the algorithm presented in this paper only once, both the reference waves (including the direct wave and the surface wave) and the ghosts can be effectively removed. The method is tested on a layered elastic earth model. The results indicate its effectiveness for reducing the ground roll and ghosts at the same time, and without harming the up-going reflections, in preparation for on-shore processing.

INTRODUCTION

On-shore seismic exploration and processing seeks to use reflection data (the scattered wavefield) to make inferences about the subsurface. The measured total wavefield consists of the reflection data and the reference wave that contains the direct wave and the surface wave/ground roll; hence, one prerequisite is to separate the reference wave and scattered wave. Filtering methods are typically employed to remove the reference wave, particularly the ground roll. That can be at the expense of damaging reflection data when ground roll is interfering with the scattered wavefield.

In addition, for buried sources and receivers, not only up-going waves are in the reflection data but also ghosts, whose existence can cause notches in the spectrum. Thus, removing the ghosts from the reflection data is another prerequisite. In this study, we will assume the source is located slightly above the air/earth surface (could be infinitely close, or on the air/earth surface), and the receivers are slightly beneath the air/earth surface. Therefore, there are receiver ghosts but no source ghosts in our study.

As a flexible and useful tool, Green's theorem provides a method to satisfy both prerequisites; i.e., removing the reference wave without damaging the reflection data and removing the ghosts from the reflection data without destroying the up-going reflected data. The distinct advantages of applying the method based on Green's theorem in off-shore plays have been demonstrated by Weglein et al. (2002); Zhang (2007); Mayhan et al. (2011); Mayhan and Weglein (2013); Tang et al. (2013); Yang et al. (2013).

Basically, wave separation from Green's theorem has a model of the world that consists of the **reference medium** and the **sources**. The choice of reference medium is arbitrary, and the choice of reference will determine what the sources have to be to arrange for the reference medium and sources together to correspond to the actual medium and experiment (Weglein et al., 2003). For on-shore plays, Green's theorem wave separation method is applicable for data either in displacement space (Pao and Varatharajulu, 1976; Weglein and Secrest, 1990) or in the PS space (Wu and Weglein, 2014). In this paper, for data in displacement space, we choose a homogeneous elastic whole space as the reference, then both the reference wave and receiver ghosts can be removed in one step while applying the elastic Green's theorem wave separation algorithm. In a companion paper (Wu and Weglein, 2015b), and for data in the PS space, the reference medium is chosen to be composed of two homogenous half-spaces, an air/acoustic half-space over an elastic half-space, then Green's theorem method can extinguish the reference wave (including the ground roll) without harming the reflection data. After obtaining the reflection data, Green's theorem provides a reflection data deghosting algorithm with a choice of a whole-space homogenous elastic reference (Wu and Weglein, 2015b).

DESCRIPTION OF THE MODEL: REFERENCE MEDIUM + SOURCES

As shown in Figure 1, the model consists of an air half-space and an elastic-earth half-space. Receivers are buried in the earth, and the active source in the form of a vertical force is applied on the free surface (F.S.). Therefore, ghosts exist at the receiver side only. The measurement surface (M.S.) can be infinitely close to the free surface, like on-surface acquisition, or several meters below the free surface, like buried-receiver acquisition; however, the receivers are coupled with the elastic medium in both situations.

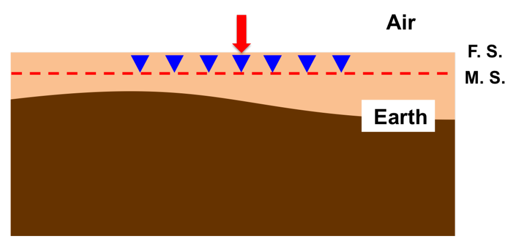

Figure 1: A generic model describing the land experiment

In this paper, we will assume that the portion of earth along the measurement surface is homogeneous and known. Within this assumption, we choose the reference medium to be a homogenous elastic whole space, as shown in Figure 2, whose property agrees with the actual earth along the measurement surface.

There are three sources acting on the homogeneous reference medium that is described in Figure 2. As shown in Figure 3, one is the active source (or the vertical force S_1) and the other two are passive sources (or the perturbations S_2 and S_3) on two sides of the measurement surface, respectively. S_1 produces the direct waves. S_2 produces the ground roll; it also produces

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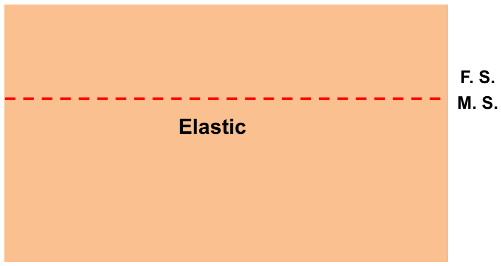

Figure 2: A homogeneous elastic whole-space reference medium

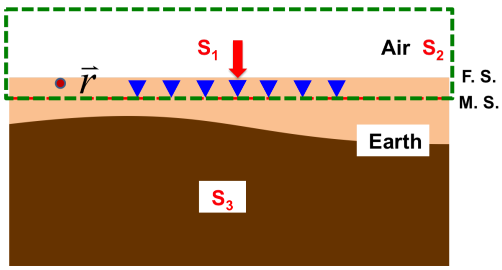

Figure 3: Three sources are acting on the reference medium that is depicted in Figure 2, and surface integral along the measurement surface will move out the contribution from S_1 and S_2 inside the enclosed the surface.

the ghosts by transferring the up-going waves that are propagating from the earth, to the down-going ones. S_3 generates up-going waves from the earth. All of these three sources contribute to providing the actual total wavefield, and the up waves due to S_3 are expected to be separated from the waves caused by both S_1 and S_2.

ELASTIC GREEN'S THEOREM WAVE SEPARATION THEORY

Background of 2D elastic wave theory

The wave equation for a 2D elastic isotropic medium is

$$\nabla \cdot \tau(\mathbf{r},\omega) + \rho\omega^2 \mathbf{u}(\mathbf{r},\omega) = \mathbf{f}(\mathbf{r},\omega), \tag{1}$$

where:

$$\tau = \lambda \nabla \cdot \mathbf{u}\, \mathbf{I} + \mu \left(\nabla \mathbf{u} + \mathbf{u}\nabla\right). \tag{2}$$

$\mathbf{u} = \begin{pmatrix} u_x \\ u_z \end{pmatrix}$ is the displacement, the 2^{nd} order tensor $\tau = \begin{pmatrix} \tau_{xx} & \tau_{xz} \\ \tau_{zx} & \tau_{zz} \end{pmatrix}$ is the stress, $\mathbf{f} = \begin{pmatrix} f_x \\ f_z \end{pmatrix}$ is the source, λ and μ are Lamé's parameters, and ρ is the density.

The impulse response of the reference medium can be written as

$$\nabla \cdot \Sigma_0(\mathbf{r},\omega) + \rho_0\omega^2 \mathbf{G}_0(\mathbf{r},\omega) = \delta(\mathbf{r})\,\mathbf{I}, \tag{3}$$

where:

$$\Sigma_{0ijk} = \lambda_0 \partial_m G_{0mk}\delta_{ij} + \mu_0(\partial_i G_{0jk} + \partial_j G_{0ik}), \qquad i,j,k = x,z. \tag{4}$$

The 2^{nd} order tensor $\mathbf{G}_0 = \begin{pmatrix} G_{0xx} & G_{0xz} \\ G_{0zx} & G_{0zz} \end{pmatrix}$ is the Green's displacement tensor, the 3^{rd} order tensor Σ_0 is the Green's stress tensor, and the source term consists of a diagonal matrix.

Elastic Green's theorem wave separation algorithm in (x,ω) domain

As seen in Figure 3, applying Green's theorem, the integral, that is along the closed semi-infinite surface lower bounded by the measurement surface, will separate the portion of the wavefield that is inside the enclosed volume due to S_3 that is outside the volume.

Starting from Equation 1 and Equation 3, and using the Green's theorem, we can obtain the wave separation algorithm (see Wu and Weglein (2015a) for the detailed derivation); i.e., we can extract the up-going waves generated by the source S_3.

In (x,ω) domain, the formula is

$$\mathbf{u}^{up}(\mathbf{r},\omega) = -\int_{m.s.} [(\hat{n}' \cdot \tau(\mathbf{r}',\omega)) \cdot \mathbf{G}_0(\mathbf{r}',\mathbf{r},\omega) - \mathbf{u}(\mathbf{r}',\omega) \cdot (\hat{n}' \cdot \Sigma_0(\mathbf{r}',\mathbf{r},\omega))]d\mathbf{r}', \tag{5}$$

where $\hat{n}'$ is the normal outside vector along the surface.

On the measurement surface, $\mathbf{r}' = (x', z' = \varepsilon_g)$,

$$\tau(x',z'=\varepsilon_g,\omega) = \lambda_0 \nabla' \cdot \mathbf{u}(x',z'=\varepsilon_g,\omega)\,\mathbf{I} + \mu_0(\nabla'\mathbf{u}(x',z'=\varepsilon_g,\omega) + \mathbf{u}(x',z'=\varepsilon_g,\omega)\nabla'), \tag{6}$$

where ε_g is the receiver's depth. Since the properties along the measurement surface have been assumed to be homogeneous in the paper, λ_0 and μ_0 are constants along the measurement surface. The choice of the reference medium depends on these invariant parameters.

Applying Equation 5 in the (x,ω) domain, we can remove the reference wave (particularly the ground roll) and the ghosts simultaneously. There is no assumption about the shape of the measurement surface; i.e., it can be either flat or rugose.

Elastic Green's theorem wave separation algorithm in (k_x,ω) Domain

If the measurement surface is horizontal and flat, then $\hat{n}' = (0,1)$. Equation 5 can be Fourier transformed to (k_x,ω) domain.

Equation 5 is expanded to be

$$\begin{aligned} u_x^{up}(\mathbf{r},\omega) &= -\int_{m.s.} [\tau_{zx}(\mathbf{r}',\omega)G_{0xx}(\mathbf{r}',\mathbf{r},\omega) + \tau_{zz}(\mathbf{r}',\omega)G_{0zx}(\mathbf{r}',\mathbf{r},\omega) \\ &\quad - u_x(\mathbf{r}',\omega)\Sigma_{0zxx}(\mathbf{r}',\mathbf{r},\omega) - u_z(\mathbf{r}',\omega)\Sigma_{0zzx}(\mathbf{r}',\mathbf{r},\omega)]dx', \\ u_z^{up}(\mathbf{r},\omega) &= -\int_{m.s.} [\tau_{zx}(\mathbf{r}',\omega)G_{0xz}(\mathbf{r}',\mathbf{r},\omega) + \tau_{zz}(\mathbf{r}',\omega)G_{0zz}(\mathbf{r}',\mathbf{r},\omega) \\ &\quad - u_x(\mathbf{r}',\omega)\Sigma_{0zxz}(\mathbf{r}',\mathbf{r},\omega) - u_z(\mathbf{r}',\omega)\Sigma_{0zzz}(\mathbf{r}',\mathbf{r},\omega)]dx'. \end{aligned} \tag{7}$$

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With reciprocity,

$$G_{0ij}(\mathbf{r}',\mathbf{r},\omega) = G_{0ji}(\mathbf{r},\mathbf{r}',\omega), i,j,k = x,z. \quad (8)$$

Applying Fourier transform over x in Equation 7 with $\int e^{-ik_x x}dx$, it will become

$$\begin{aligned}
&\tilde{u}_x^{up}(k_x,z,\omega)\\
&= -[\tilde{\tau}_{zx}(k_x,z',\omega)\tilde{G}_{0xx}(k_x,z,z',\omega) + \tilde{\tau}_{zz}(k_x,z',\omega)\tilde{G}_{0xz}(k_x,z,z',\omega)\\
&- \tilde{u}_x(k_x,z',\omega)\tilde{\Sigma}_{0zxx}(k_x,z',z,\omega) - \tilde{u}_z(k_x,z',\omega)\tilde{\Sigma}_{0zzx}(k_x,z',z,\omega)]|_{z'=\varepsilon_g},\\
&\tilde{u}_z^{up}(k_x,z,\omega)\\
&= -[\tilde{\tau}_{zx}(k_x,z',\omega)\tilde{G}_{0zx}(k_x,z,z',\omega) + \tilde{\tau}_{zz}(k_x,z',\omega)\tilde{G}_{0zz}(k_x,z,z',\omega)\\
&- \tilde{u}_x(k_x,z',\omega)\tilde{\Sigma}_{0zxz}(k_x,z',z,\omega) - \tilde{u}_z(k_x,z',\omega)\tilde{\Sigma}_{0zzz}(k_x,z',z,\omega)]|_{z'=\varepsilon_g},
\end{aligned} \quad (9)$$

where tildes represent the terms in k_x domain, and z' is evaluated at the receiver's depth ε_g. Specifically,

$$\begin{aligned}
&\tilde{\Sigma}_{0zxx}(k_x,z',z,\omega)\\
&=\mu_0[\partial_{z'}\tilde{G}_{0xx}(k_x,z,z',\omega) - ik_x\tilde{G}_{0xz}(k_x,z,z',\omega)],\\
&\tilde{\Sigma}_{0zzx}(k_x,z',z,\omega)\\
&=\gamma_0\partial_{z'}\tilde{G}_{0xz}(k_x,z,z',\omega) - \lambda_0(ik_x)\tilde{G}_{0xx}(k_x,z,z',\omega),\\
&\tilde{\Sigma}_{0zxz}(k_x,z',z,\omega)\\
&=\mu_0[\partial_{z'}\tilde{G}_{0zx}(k_x,z,z',\omega) - ik_x\tilde{G}_{0zz}(k_x,z',z,\omega)],\\
&\tilde{\Sigma}_{0zzz}(k_x,z',z,\omega)\\
&=\gamma_0\partial_{z'}\tilde{G}_{0zz}(k_x,z,z',\omega) - \lambda_0(ik_x)\tilde{G}_{0zx}(k_x,z,z',\omega),
\end{aligned} \quad (10)$$

where γ_0 is the bulk modulus, and $\gamma_0 = \lambda_0 + 2\mu_0$.

For a reference medium as homogenous elastic whole space, both the Green's displacement tensor and its stress tensor can be expressed analytically (see appendix A for $\mathbf{G}_0$).

It is deserving emphasis that applying the algorithm in (k_x,ω) domain, we can locate the output point $\mathbf{r}$ on the measurement surface to be part of the volume above, to extract the the up-going wavefield that is portion of the actually measured data.

NUMERICAL EVALUATION

We test the (k_x,ω) domain wave separation algorithm on a two layered elastic earth model, as seen in Figure 4. A vertical force $(0,F_z)$ is applied on the free surface, and receivers are buried at depth 30m. For simplicity, the space above the free surface is set to be vacuum. The properties of the earth are listed in Table 1. The output point $\mathbf{r}$ is arranged to be on the measurement surface and treated as part of the volume above.

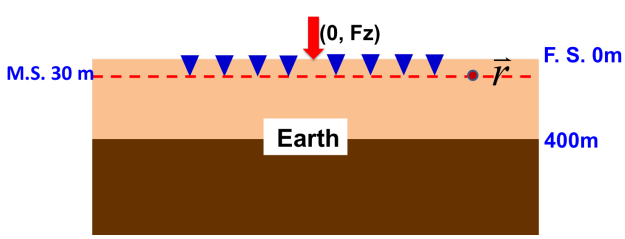

Figure 4: A two layered elastic earth model for numeric test

Layer's Number	P Velocity (m/s)	S Velocity (m/s)	Density (kg/m^3)
1	1800	1200	1500
2	4000	2500	1800

Table 1: The parameters of the earth model in Figure 4

The trace interval is 2m, the maximum offset is 3000m, the time sampling interval is 4ms, and the total time sampling length is 3s. Then the data of displacement **u** with both x and z components are generated using analytic forms. As shown in Figure 5(a) and Figure 5(d), the data have strong Rayleigh wave and relatively weak scattered wave. Besides, the ghosts are interfering with the up-going waves. All the figures are in the same scales.

Putting the multicomponent data into the wave separation formula of Equation 9, the up-going waves can be separated, with x and z components. To evaluate the accuracy of the results, we analytically create the data consisting of only the up waves, with the given model of Figure 4. These data will play as criteria to examine the calculation results. The x component of separated up waves from Green's theorem separation algorithm (Figure 5(b)) is compared with the x component created up waves with analytic form (Figure 5(c)), and the comparison shows that both their amplitudes and phases match very well. The conclusion is the same for the comparison of z components (see Figure 5(e) and Figure 5(f)). As the results shown in Figure 5, both the Rayleigh waves and ghosts are extinguished, and there is no harm to the up-going reflection data.

DISCUSSION OF SELECTING THE REFERENCE MEDIUM

As described in the introduction, we have two strategies to remove the reference waves (including the direct wave and the surface wave) and the ghosts. One is removing both of them at the same time by choosing a homogenous elastic whole space as the reference medium. The other is first removing the reference waves by choosing a two-half-space reference medium that is a half-space of homogenous air over a half-space of homogenous elastic earth, then deghosting with the reference medium to be a whole space of homogeneous elastic. Conceptually, it's applicable either way; however, based on our present study, the Green's function in displacement space is complicated for a discontinuous medium, especially for the situation that both the source and receiver points of Green's function are close to the air-elastic boundary/free surface. Practically, the wave separation formula with such a complicated Green's function may produce an unstable result. That's the reason why we select the first strategy that uses a Green's function with a simpler analytic form. That strategy provides a stable and useful approach for on-shore preprocessing in displacement space.

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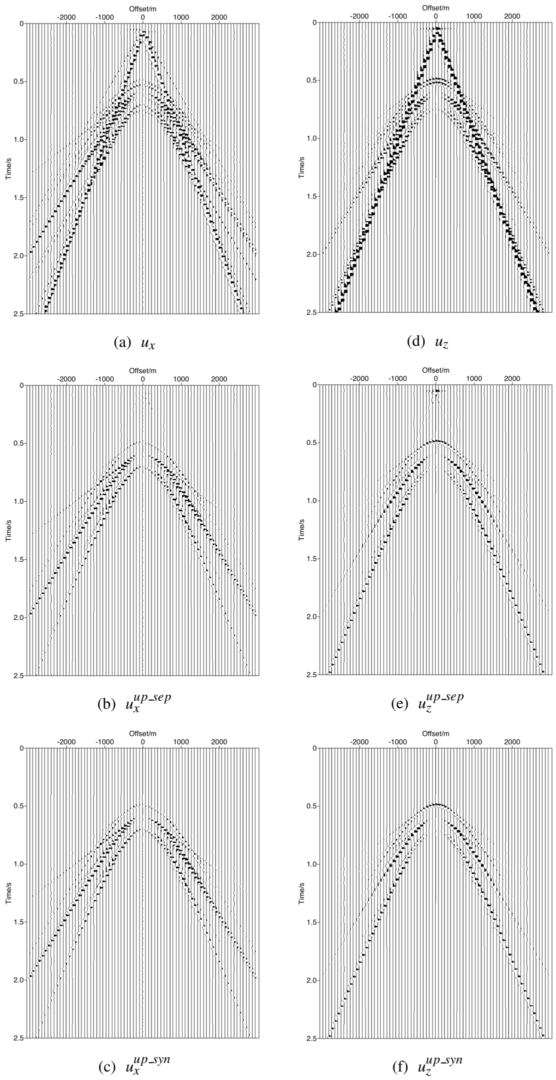

(a) u_x (d) u_z

(b) $u_x^{up\_sep}$ (e) $u_z^{up\_sep}$

(c) $u_x^{up\_syn}$ (f) $u_z^{up\_syn}$

Figure 5: Wave separation results. (a) is x component total wave; (b) is the x component separated up wave; (c) is the x component created up wave with analytic form; (d) is z component total wave; (e) is the z component separated up wave; (f) is the z component created up wave with analytic form.

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CONCLUSION

From the theoretic derivation and numeric test in this paper, the elastic Green's theorem based wave separation method in displacement space has the potential to remove both the ground roll and the ghosts from on-shore data. In addition, by choosing a homogenous elastic whole space as reference, we can remove these two waves simultaneously. The algorithm that we develop in this paper has two requirements: (1) both the displacement and the traction (or the derivative of displacement) along the measurement surface; and (2) the homogeneous and known properties along the measurement surface. Our interest in reducing the demanding on-shore data collection and requiring near surface properties motivates the next steps in our research: (1) altering the Green's function with a Dirichlet boundary condition to reduce the requirement of traction; (2) pursuing an alternative approach for these wave separation objectives without the requirement of known properties along the measurement surface.

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APPENDIX A: ANALYTIC FORM OF THE GREEN'S FUNCTION

For a whole space homogeneous elastic medium, the Green's function in displacement space can be expressed analytically as

$$\mathbf{G}_0(\mathbf{r}',\mathbf{r},\omega) = \frac{1}{2\pi}\int \begin{pmatrix} \tilde{G}_{xx}(k_x,z',z,\omega) & \tilde{G}_{xz}(k_x,z',z,\omega) \\ \tilde{G}_{zx}(k_x,z',z,\omega) & \tilde{G}_{zz}(k_x,z',z,\omega) \end{pmatrix} e^{ik_x(x'-x)}dk_x, \tag{A-1}$$

where

$$\begin{aligned}
&\tilde{G}_{xx}(k_x,z',z,\omega) \\
&= \frac{1}{\rho_0\omega^2}\left(k_x^2\frac{e^{i\nu_0|z'-z|}}{2i\nu_0} + \eta_0^2\frac{e^{i\eta_0|z'-z|}}{2i\eta_0}\right), \\
&\tilde{G}_{xz}(k_x,z',z,\omega) \\
&= \frac{1}{\rho_0\omega^2}\left(k_x\nu_0 sgn(z'-z)\frac{e^{i\nu_0|z'-z|}}{2i\nu_0} - k_x\eta_0 sgn(z'-z)\frac{e^{i\eta_0|z'-z|}}{2i\eta_0}\right), \\
&\tilde{G}_{zx}(k_x,z',z,\omega) \\
&= \frac{1}{\rho_0\omega^2}\left(k_x\nu_0 sgn(z'-z)\frac{e^{i\nu_0|z'-z|}}{2i\nu_0} - k_x\eta_0 sgn(z'-z)\frac{e^{i\eta_0|z'-z|}}{2i\eta_0}\right), \\
&\tilde{G}_{zz}(k_x,z',z,\omega) \\
&= \frac{1}{\rho_0\omega^2}\left(\nu_0^2\frac{e^{i\nu_0|z'-z|}}{2i\nu_0} + k_x^2\frac{e^{i\eta_0|z'-z|}}{2i\eta_0}\right),
\end{aligned} \tag{A-2}$$

and

$$\nu_0 = \begin{cases} \sqrt{k_{\alpha_0}^2 - k_x^2} & \text{if } k_x < k_{\alpha_0} \\ i\sqrt{k_x^2 - k_{\alpha_0}^2} & \text{if } k_x > k_{\alpha_0} \end{cases} \qquad k_{\alpha_0} = \frac{\omega}{\alpha_0},$$

$$\eta_0 = \begin{cases} \sqrt{k_{\beta_0}^2 - k_x^2} & \text{if } k_x < k_{\beta_0} \\ i\sqrt{k_x^2 - k_{\beta_0}^2} & \text{if } k_x > k_{\beta_0} \end{cases} \qquad k_{\beta_0} = \frac{\omega}{\beta_0}.$$

α_0 and β_0 are P wave velocity and S wave velocity in the medium, respectively.

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