Imaging the wavefield at depth without the velocity ... forward and inverse diagrams point the way

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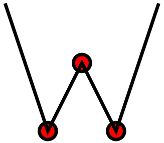
Main points of the talk

- The calculation and imaging of the wavefield at depth using the model type independent part of the ISS
- Unknown medium implies one frequency at depth requires all frequencies on the surface

Outline

Motivation and approach

- Generalize the imaging algorithm to one independent of model type
- Use the lessons learned from internal multiple algorithm
- Take the model type independent contributions from V through specific diagrams[1] [2] [3]



[1] Araujo F. V., *Linear and non-linear methods derived from scattering theory: backscattered tomography and internal multiple attenuation*, PhD thesis, Universidade Federal da Bahia, 1994

[2] Weglein et al, *An inverse scattering series method for attenuating multiples in seismic reflection data*, Geophysics 62 (1997), pp. 1975-1989

[3] Weglein et al, *Inverse scattering series and seismic exploration*, Topical Review Inverse Problems 19 (2003), pp. R27-R83

Motivation and approach

- **Imaging diagrams** [4 5 6]

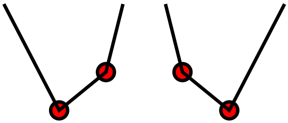


Figure: ***Second order diagrams for imaging primaries***

- **The model type independent part of *V* is placed in diagrams for imaging primaries**
- **Determine the usefulness of this approach**

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[4] Shaw S.A., *An inverse scattering series algorithm for depth imaging of reflection data from a layered acoustic medium with an unknown velocity model*, (2005) Ph.D. Thesis, University of Houston

[5] Liu F., *Multi-dimensional depth imaging without an adequate velocity model*, (2006) Ph.D. Thesis, University of Houston

[6] Zhang H., *Direct non-linear acoustic and elastic inversion: towards fundamentally new comprehensive and realistic target identification*, (2006) Ph.D. Thesis, University of Houston

Forward scattering series

- **Wave equations in actual and reference media**

$$LG = -I$$

$$L_0 G_0 = -I$$

- **Lippmann–Schwinger Eq – valid everywhere**

$$\psi_s = G - G_0 = G_0 V G$$

- **Forward Scattering Series – valid everywhere**

$$\psi_s \equiv G - G_0 = G_0 V G_0 + G_0 V G_0 V G_0 + \ldots$$

- **Data $=$ scattered field on the MS**

$$D = (\psi_s)_{MS}$$

Series for the perturbation and the wavefield at depth

- Series for the perturbation and the wavefield at depth in orders of the data

$$V = V_1 + V_2 + V_3 + \dots$$

$$\psi_s = \psi_s^1 + \psi_s^2 + \psi_s^3 + \dots$$

- Plug these into the FSS and equate like orders in data from both sides

$$\begin{aligned}
\psi_s^1 &= G_0 V_1 G_0 \\
\psi_s^2 &= G_0 V_2 G_0 + G_0 V_1 G_0 V_1 G_0 \\
\psi_s^3 &= G_0 V_3 G_0 + G_0 V_2 G_0 V_1 G_0 + G_0 V_1 G_0 V_2 G_0 \\
&\quad + G_0 V_1 G_0 V_1 G_0 V_1 G_0 \\
&\vdots
\end{aligned}$$

Inverse scattering series

- On the measurement surface

$$D = (G_0 V_1 G_0)_{ms}$$
$$0 = (G_0 V_2 G_0 + G_0 V_1 G_0 V_1 G_0)_{ms}$$
$$0 = (G_0 V_3 G_0 + G_0 V_2 G_0 V_1 G_0 + G_0 V_1 G_0 V_2 G_0 + G_0 V_1 G_0 V_1 G_0 V_1 G_0)_{ms}$$
$$\vdots$$

- Inverse scattering series

$$V = V_1 + V_2 + V_3 + \ldots$$

- History and description of subseries method [7]

[7] Weglein et al, *Inverse scattering series and seismic exploration*, Topical Review Inverse Problems, 19 (2003), pp. R27-R83

The wavefield at depth[8]

- **Step 1: calculate the perturbation operator**

$$D = (G_0 V_1 G_0)_{ms}$$
$$0 = (G_0 V_2 G_0 + G_0 V_1 G_0 V_1 G_0)_{ms} \quad (1)$$

- **Step 2: calculate the wavefield at depth order by order using**

$$\psi_s^1 = G_0 V_1 G_0$$
$$\psi_s^2 = G_0 V_2 G_0 + G_0 V_1 G_0 V_1 G_0 \quad (2)$$

- **Step 3: Sum all the orders up**

$$\psi_s = \psi_s^1 + \psi_s^2 + \dots$$

[8] Weglein et al 2000, *Imaging and inversion at depth without a velocity model: theory, concepts and initial evaluation*, SEG 2000 Expanded Abstracts.

Homogeneous acoustic Green's functions

- **Homogeneous acoustic wave equation**

$$\nabla^2\phi(x,t) - \frac{1}{c_0^2}\frac{\partial^2\phi(x,t)}{\partial t^2} = -\delta(x)\delta(t)$$

- **Solution in the space-frequency domain**

$$G_0(x,\omega) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dq \frac{e^{ik\cdot x}}{k^2+q^2-\frac{\omega^2}{c_0^2}}$$

- **Weyl integral**

$$G_0(x;\omega) = P.V. + \frac{1}{2\pi}\int_{-\infty}^{\infty} dk_x \frac{e^{i[k(x_g-x_s)+iq|z_g-z_s|]}}{2iq}$$

- **When used in the inverse scattering series, the *P.V.* part requires information about the earth's model type.**

Alternative formulas

- **Wavenumber-frequency domain**

$$G_0(k,q,\omega) = \frac{1}{k^2 + q^2 - \frac{\omega^2}{c_0^2} - i\epsilon}$$

- **Selection of $\pm\epsilon$ based on causality**
- **Space domain**

$$G_0(x;\omega) = \left(\frac{1}{2\pi}\right)^2 \int dk \int dq \frac{e^{-ik(x_g - x_s)} e^{-iq(z_g - z_s)}}{k^2 + q^2 - \frac{\omega^2}{c_0^2} - i\epsilon}$$

- **In this formula $P.V.$ has already been discarded.**

Calculating V_1

- Use the formulas for the inverse scattering series

$$D = (G_0 V_1 G_0)_{ms}$$

- We found [9]

$$V_1(k_g, q_g, k_s, q_s, \omega) = -4 q_g q_s e^{i q_g z_g} e^{i q_s z_s} D(k_g, k_s, \omega)$$

where

$$k_g^2 + q_g^2 = \frac{\omega^2}{c_0^2}, \quad k_s^2 + q_s^2 = \frac{\omega^2}{c_0^2}$$

[9] Ramirez et al, *Note on velocity independent contributions in the inverse scattering series for processing primaries*, 2007 M-OSRP report.

Calculating V_2

- **Use the formulas for the inverse scattering series**

$$0 = (G_0 V_2 G_0 + G_0 V_1 G_0 V_1 G_0)_{ms}$$

- **We found the imaging part of V_2** [10]

$$V_2^{IM}(k_g, q_g, k_s, q_s, \omega) = \frac{2 q_g q_s e^{i(q_g z_g + q_s z_s)}}{\pi} \int_{-\infty}^{\infty} dk_\lambda q_\lambda$$

$$\left[e^{iq_\lambda(z_s - z_g)} \int_{-\infty}^{\infty} dz_1 e^{iz_1(q_g + q_\lambda)} D(k_g, k_\lambda, z_1) \int_{z_1+\epsilon}^{\infty} dz_2 e^{iz_2(-q_\lambda + q_s)} D(k_\lambda, k_s, z_2) \right.$$

$$\left. + e^{iq_\lambda(z_g - z_s)} \int_{-\infty}^{\infty} dz_3 e^{iz_3(q_g - q_\lambda)} D(k_g, k_\lambda, z_3) \int_{-\infty}^{z_3-\epsilon} dz_4 e^{iz_4(q_\lambda + q_s)} D(k_\lambda, k_s, z_4) \right]$$

where

$$k_g^2 + q_g^2 = \frac{\omega^2}{c_0^2}, \quad k_s^2 + q_s^2 = \frac{\omega^2}{c_0^2}, \quad k_\lambda^2 + q_\lambda^2 = \frac{\omega^2}{c_0^2}$$

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[10] Ramirez et al, *Note on velocity independent contributions in the inverse scattering series for processing primaries*, 2007 M-OSRP report.

First order wavefield at depth

- Use the formulas for the wavefield at depth

$$\psi_s^1 = G_0 V_1 G_0$$

- We found

$$\psi_s^1(k_g, q_1, k_s, q_2; \omega_1) = \frac{-4 q_g q_s e^{i q_g z_g} e^{i q_s z_s} D(k_g, k_s, \omega)}{\left(k_g^2 + q_1^2 - \frac{\omega_1^2}{c_0^2} - i\epsilon\right)\left(k_s^2 + q_2^2 - \frac{\omega_1^2}{c_0^2} - i\epsilon\right)}$$

$$k_g^2 + q_1^2 = \frac{\omega_1^2}{c_0^2}, \quad k_s^2 + q_2^2 = \frac{\omega_1^2}{c_0^2}$$

$$k_g^2 + q_g^2 = \frac{\omega^2}{c_0^2}, \quad k_s^2 + q_s^2 = \frac{\omega^2}{c_0^2}$$

$$\frac{\omega_1^2}{c_0^2} = \frac{\left[(q_g + q_s)^2 + k_g^2 + k_s^2\right]^2 - 4 k_g^2 k_s^2}{4 (q_g + q_s)^2}$$

Second order wavefield at depth

- **Use the formulas for the wavefield at depth**

$$\psi_s^2 = G_0 V_2 G_0 + G_0 V_1 G_0 V_1 G_0$$

- **We found**

$$\psi_s^{2IM}(k_g, q_1, k_s, q_2; \omega_1) = \frac{2}{\pi} \frac{q_g e^{i q_g z_g}}{k_g^2 + q_1^2 - \frac{\omega_1^2}{c_0^2} - i\epsilon} \frac{q_s e^{i q_s z_s}}{k_s^2 + q_2^2 - \frac{\omega_1^2}{c_0^2} - i\epsilon} \int dk_\lambda \frac{q_\lambda}{q_{\lambda_1}} (q_{\lambda_1} - q_\lambda)$$

$$\left[e^{i q_\lambda (z_s - z_g)} \int_{-\infty}^{\infty} dz_1 e^{i z_1 (q_g + q_\lambda)} D(k_g, k_\lambda, z_1) \int_{z_1+\epsilon}^{\infty} dz_2 e^{i z_2 (-q_\lambda + q_s)} D(k_\lambda, k_s, z_2) \right.$$

$$\left. + e^{i q_\lambda (z_g - z_s)} \int_{-\infty}^{\infty} dz_3 e^{i z_3 (q_g - q_\lambda)} D(k_g, k_\lambda, z_3) \int_{-\infty}^{z_3-\epsilon} dz_4 e^{i z_4 (q_\lambda + q_s)} D(k_\lambda, k_s, z_4) \right]$$

$$k_g^2 + q_1^2 = \frac{\omega_1^2}{c_0^2}, \quad k_s^2 + q_2^2 = \frac{\omega_1^2}{c_0^2}, \quad k_\lambda^2 + q_{\lambda_1}^2 = \frac{\omega_1^2}{c_0^2}$$

$$k_g^2 + q_g^2 = \frac{\omega^2}{c_0^2}, \quad k_s^2 + q_s^2 = \frac{\omega^2}{c_0^2}, \quad k_\lambda^2 + q_\lambda^2 = \frac{\omega^2}{c_0^2}$$

$$\frac{\omega_1^2}{c_0^2} = \frac{\left[(q_g + q_s)^2 + k_g^2 + k_s^2 \right]^2 - 4 k_g^2 k_s^2}{4 (q_g + q_s)^2}$$

Total wavefield at depth

- **Summing 1st and 2nd orders we find**

$$\psi_s^{2nd}(k_g,q_1,k_s,q_2;\omega_1)=\frac{-4q_gq_se^{iq_gz_g}e^{iq_sz_s}}{\left(k_g^2+q_1^2-\frac{\omega_1^2}{c_0^2}-i\epsilon\right)\left(k_s^2+q_2^2-\frac{\omega_1^2}{c_0^2}-i\epsilon\right)}$$

$$\Bigg\{D(k_g,z_g,k_s,z_s,\omega)+\frac{1}{2\pi}\int_{-\infty}^{\infty}dk_\lambda\frac{q_\lambda}{q_{\lambda_1}}(q_\lambda-q_{\lambda_1})$$

$$\Bigg[e^{iq_\lambda(z_s-z_g)}\int_{-\infty}^{\infty}dz_1e^{iz_1(q_g+q_\lambda)}D(k_g,k_\lambda,z_1)\int_{z_1+\epsilon}^{\infty}dz_2e^{iz_2(-q_\lambda+q_s)}D(k_\lambda,k_s,z_2)$$

$$+e^{iq_\lambda(z_g-z_s)}\int_{-\infty}^{\infty}dz_3e^{iz_3(q_g-q_\lambda)}D(k_g,k_\lambda,z_3)\int_{-\infty}^{z_3-\epsilon}dz_4e^{iz_4(q_\lambda+q_s)}D(k_\lambda,k_s,z_4)\Bigg]\Bigg\}$$

$$k_g^2+q_1^2=\frac{\omega_1^2}{c_0^2},\quad k_s^2+q_2^2=\frac{\omega_1^2}{c_0^2},\quad k_\lambda^2+q_{\lambda_1}^2=\frac{\omega_1^2}{c_0^2}$$

$$k_g^2+q_g^2=\frac{\omega^2}{c_0^2},\quad k_s^2+q_s^2=\frac{\omega^2}{c_0^2},\quad k_\lambda^2+q_\lambda^2=\frac{\omega^2}{c_0^2}$$

$$\frac{\omega_1^2}{c_0^2}=\frac{\left[(q_g+q_s)^2+k_g^2+k_s^2\right]^2-4k_g^2k_s^2}{4(q_g+q_s)^2}$$

Imaging the wavefield

- **We calculated**

$$\psi_s^{2nd}(k_g, q_1, k_s, q_2; \omega_1)$$

- **Imaging step**

$$I(k_g, k_s, z) = \int d\omega_1 e^{ik_z z} \psi_s^{2nd}(k_g, q_1, k_s, q_2; \omega_1)$$

- **Transform over the horizontal wavenumbers**

$$I(x, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d(k_g - k_s) e^{-i(k_g - k_s)x} I(k_g, k_s, z)$$

Summary

- The calculation and imaging of the wavefield at depth using the model type independent part of ISS.
- Diagrams show the way.
- Unknown medium implies one frequency at depth requires all frequencies on the surface.
- Further tests are needed to verify the usefulness of the theory.

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Petrobras

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PetroChina

PGS
PGS
Petroleum Geo-Services

ارامكو السعودية
Saudi Aramco
Saudi Aramco

Shell

STATOIL
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