Wavelet estimation for towed streamer data

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Abstract

Weglein and Secrest (1990) present a method for computing the scattered wavefield between the measurement surface and free surface, and the reference wavefield below the measurement surface, given both the pressure and its normal derivative along the cable. Osen et al. (1998) and Tan (1992) show that the wavelet due to an isotropic source can be determined from pressure on the measurement surface and an extra hydrophone between the measurement surface and the free surface. Tan (1999) observes that in practice it is possible to well-estimate the wavefield above single towed streamer for points not directly under the source. Using the Tan (1999) wavefield prediction the wavelet can in principle be estimated from only a single cable (Weglein et al. 2000). However, the integral required for wavelet estimation requires data along the cable including the region excluded by the Tan's prediction. An approach to addressing that problem is presented here that adopts a generalized inverse viewpoint to find a well estimated approximation to the wavelet. First tests are encouraging and reported here; others are planned.

In this paper we will review the theory and show a new approach to wavelet estimation below the cable from only a single cable. Further work will focus on application to deghosting and multiple attenuation.

1 Introduction

In wave-theoretic multiple attenuation methods (eg. Carvalho, 1992, Weglein et al. 1997, Verschuur 1992), knowledge of the source wavelet is one of the requirements. The energy-minimization criterion is often applied in practice to estimate the wavelet. Current methods based on the energy minimization criterion have proven to be useful under many circumstances. However, under complex conditions, e.g., weak internal multiples proximal to weak subsalt primaries, experience suggests that the energy minimization criterion is too blunt an instrument for that degree of subtlety. This is the motivation for deriving new methods to provide the source wavelet. The industry trend towards complex and costly plays raises the bar of required effectiveness for wave theoretic multiple removal and imaging-inversion techniques, and the prerequisites, such as the wavelet that needs to be provided.

The goal of the research described here is to test and progress the development of new wavelet estimation methods that can be estimated from only the pressure on the cable. In the following we will first discuss extinction theorem; then we show how to predict the normal derivatives of the wavefield above the measurement surface.

2 Extinction Theorem

The acoustic wave equation can be written in the following form in the frequency domain, where $\mathbf{r}'$ is any point in a half space below the free surface, $\mathbf{r}_0$ is the source location, $A(\omega)$ is the source signature, ω is the angular frequency, c is the actual velocity, and P is the pressure field.

$$\nabla^2 P\left(\mathbf{r}', \mathbf{r}_0, \omega\right) + \frac{\omega^2}{c^2\left(\mathbf{r}'\right)} P\left(\mathbf{r}', \mathbf{r}_0, \omega\right) = A\left(\omega\right) \delta\left(\mathbf{r}' - \mathbf{r}_0\right) \tag{1}$$

Using scattering theory, the actual earth can be parameterized as a homogeneous velocity reference medium with embedded reflectors. Hence we replace c with c_0

$$\frac{1}{c^2\left(\mathbf{r}'\right)} = \frac{1}{c_0}\left[1 - \alpha\left(\mathbf{r}'\right)\right] \tag{2}$$

where c_0 is the reference medium velocity, and $\alpha\left(\mathbf{r}'\right)$ is called the scattered index, which is used to characterize the difference between the actual and reference media. Considering the Green's function in a homogenous medium with Dirichlet boundary conditions at both the free surface and the measurement surface due to point source at $\mathbf{r}$, such that

$$\nabla^2 G_0^{DD}\left(\mathbf{r}, \mathbf{r}', \omega\right) + \frac{\omega^2}{c_0^2} G_0^{DD}\left(\mathbf{r}, \mathbf{r}', \omega\right) = \delta\left(\mathbf{r} - \mathbf{r}'\right) \tag{3}$$

where $G_0^{DD}\left(\mathbf{r}, \mathbf{r}', \omega\right)$ is a 2-D Green's function, which can be obtained by the method of images based on Poisson sum formula (Morse and Feshbach, 1953, chapter 7 vol. 1). $G_0^{DD}\left(\mathbf{r}, \mathbf{r}', \omega\right)$ converges rapidly in a numerical implementation of the following expression.

$$G_0^{DD}(\mathbf{r}, \mathbf{r}', \omega) = \frac{4\pi i}{H} \sum_{n=1}^{\infty} \sin\left(\frac{\pi z}{H} n\right) \sin\left(\frac{\pi z'}{H} n\right) \frac{1}{\beta} e^{i\beta|x - x'|}$$

where $\mathbf{r}'$ represents a source location, $\mathbf{r}$ represents a receiver location, H is the depth of M.S., and β , which controls whether or not the Green's function is decaying exponentially or travel horizontally, is defined as following

$$\beta = \sqrt{(\frac{\omega}{c_0})^2 - (\frac{n\pi}{H})^2}$$

Applying Green's theorem to equations (1) and (3)

$$\iint_S d\mathbf{s}' \left[P(\mathbf{r}', \mathbf{r}_0, \omega) \frac{\partial G_0^{DD}(\mathbf{r}, \mathbf{r}', \omega)}{\partial \mathbf{n}'} - G_0^{DD}(\mathbf{r}, \mathbf{r}', \omega) \frac{\partial P(\mathbf{r}', \mathbf{r}_0, \omega)}{\partial \mathbf{n}'} \right]$$

$$= \iiint_v d\mathbf{r}' \left[P(\mathbf{r}', \mathbf{r}_0, \omega) \nabla^2 G_0^{DD}(\mathbf{r}, \mathbf{r}', \omega) - G_0^{DD}(\mathbf{r}, \mathbf{r}', \omega) \nabla^2 P(\mathbf{r}', \mathbf{r}_0, \omega) \right] \quad (4)$$

Multiplying equation (3) by $P(\mathbf{r}', \mathbf{r}_0, \omega)$, and equation (1) by $G_0^{DD}(\mathbf{r}, \mathbf{r}', \omega)$, and then substituting them into the right hand side of equation (4), we have

$$\begin{aligned} & \iint_S d\mathbf{s}' \left[P(\mathbf{r}', \mathbf{r}_0, \omega) \frac{\partial G_0^{DD}(\mathbf{r}, \mathbf{r}', \omega)}{\partial \mathbf{n}'} - G_0^{DD}(\mathbf{r}, \mathbf{r}', \omega) \frac{\partial P(\mathbf{r}', \mathbf{r}_0, \omega)}{\partial \mathbf{n}'} \right] \\ = & \iiint_v d\mathbf{r}' P(\mathbf{r}', \mathbf{r}_0, \omega) \delta(\mathbf{r} - \mathbf{r}') \\ & + \iiint_v d\mathbf{r}' \left[-\frac{\omega^2}{c_0^2} G_0^{DD}(\mathbf{r}, \mathbf{r}', \omega) \alpha(\mathbf{r}') P(\mathbf{r}', \mathbf{r}_0, \omega) \right] \\ & \iiint_v d\mathbf{r}' \left[-A(\omega) G_0^{DD}(\mathbf{r}, \mathbf{r}', \omega) \delta(\mathbf{r}' - \mathbf{r}_0) \right] \end{aligned} \quad (5)$$

If we choose the integral volume V to be between the free surface (F.S.) and the measurement surface (M.S.), then the second term on the right hand side of equation (5) will be zero since the scatterer $\alpha(\mathbf{r}')$ (i.e. Earth) is outside of the volume V. We then choose $\mathbf{r}$ above M.S., and applying the Delta function property,

$$\iiint_v d\mathbf{r}' [\delta(\mathbf{r} - \mathbf{r}') f(\mathbf{r}')] = f(\mathbf{r})$$

we have

$$\begin{aligned} & \iint_S d\mathbf{s}' \left[P(\mathbf{r}', \mathbf{r}_0, \omega) \frac{\partial G_0^{DD}(\mathbf{r}, \mathbf{r}', \omega)}{\partial \mathbf{n}'} - G_0^{DD}(\mathbf{r}, \mathbf{r}', \omega) \frac{\partial P(\mathbf{r}', \mathbf{r}_0, \omega)}{\partial \mathbf{n}'} \right] \\ = & P(\mathbf{r}, \mathbf{r}_0, \omega) - A(\omega) G_0^{DD}(\mathbf{r}, \mathbf{r}_0, \omega) \end{aligned}$$

Finally, since we have chosen the Green's function $G_0^{DD}(\mathbf{r}, \mathbf{r}', \omega)$ to satisfy Dirichlet boundary conditions on both F.S. and M.S., then

$$P(\mathbf{r}, \mathbf{r}_0, \omega) - A(\omega)G_0^{DD}(\mathbf{r}, \mathbf{r}_0, \omega) = \iint_S ds' \left[P(\mathbf{r}', \mathbf{r}_0, \omega) \frac{\partial G_0^{DD}(\mathbf{r}, \mathbf{r}', \omega)}{\partial \mathbf{n}'} \right] \tag{6}$$

where $\mathbf{r}$ is between F.S. and M.S. (figure 1).

3 Normal derivatives

As showed above in equation (6), $G_0^{DD}(\mathbf{r}, \mathbf{r}', \omega)$ is critical in order to get the wavefield above M.S. It is a function of the frequency and the depth of the measurement surface. Tan (1999) discovered that $G_0^{DD}(\mathbf{r}, \mathbf{r}', \omega)$ is vanishingly small for typical marine streamer depths of approximately 6m and seismic frequency less than 125Hz (Figure 2). Therefore, the second term on the left hand side of equation (6) can be ignored in comparison with the other terms. Also we choose the Green's function to satisfy Dirichlet boundary conditions on both F.S. and M.S., and we assume that the pressure at F.S. will be vanishing. This results in the key observation:

$$P(\mathbf{r}, \mathbf{r}_0, \omega) \approx \iint_{MS} ds' \left[P(\mathbf{r}', \mathbf{r}_0, \omega) \frac{\partial G_0^{DD}(\mathbf{r}, \mathbf{r}', \omega)}{\partial \mathbf{n}'} \right] \tag{7}$$

This equation will be used to predict the wavefield above M.S. through an integral over the measurement surface once we obtain the Green's function, then we can compute the normal derivatives over the cable by a finite difference approximation, or by taking normal derivatives directly from equation (7).

4 Wavelet estimation

Since we require the normal derivatives under the source for wavelet estimation, we modify the idea of calculating the normal derivatives above the cable without dropping the wavelet term $A(\omega)G_0^{DD}(\mathbf{r}, \mathbf{r}', \omega)$. Hence

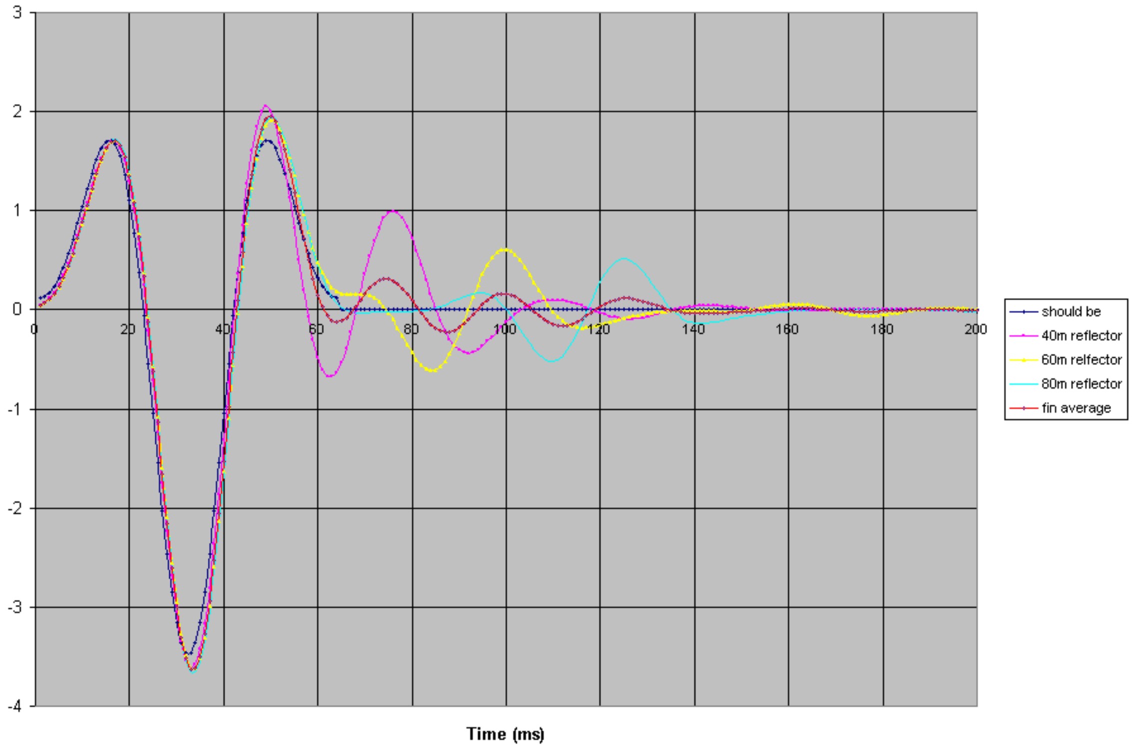

Figure 1: Wavelet estimation from water depth 40m, 60m and 80m reflector respectively. The wavelet estimated from water depth 80m reflector has least artifact; the final averaged (red) of three estimated wavelets is close to the correct one (black).

$$\frac{\partial}{\partial z}P\left(\mathbf{r},\mathbf{r}_0,\omega\right) = A(\omega)\frac{\partial}{\partial z}G_0^{DD}(\mathbf{r},\mathbf{r}_0,\omega) + \frac{\partial}{\partial z}\iint_{.M.S.} d\mathbf{r}'\left[P\left(\mathbf{r}',\mathbf{r}_0,\omega\right)\frac{\partial G_0^{DD}\left(\mathbf{r},\mathbf{r}',\omega\right)}{\partial \mathbf{n}'}\right] \tag{8}$$

If we choose $z \rightarrow \tilde{z}$, which represents the depth of M.S., we approximate

$$\frac{\partial}{\partial z}P\left(\mathbf{r},\mathbf{r}_0,\omega\right) \approx \frac{\partial}{\partial \tilde{z}}P\left(\mathbf{r},\mathbf{r}_0,\omega\right) \tag{9}$$

which will be used to estimate the normal derivatives required in wavelet estimation formula (Weglein and Secrest, 1990).

Rewriting the wavelet estimation based on the Green's function $G_0^D\left(\mathbf{r}_b,\mathbf{r}',\omega\right)$, which only satisfies the Dirichlet condition on free surface,

$$-A(\omega)G_0^D(\mathbf{r}_b,\mathbf{r}_0,\omega)=\iint_{MS} d\mathbf{s}\left[P\left(\mathbf{r},\mathbf{r}_0,\omega\right)\frac{\partial G_0^D\left(\mathbf{r}_b,\mathbf{r},\omega\right)}{\partial \mathbf{n}}-G_0^D\left(\mathbf{r}_b,\mathbf{r},\omega\right)\frac{\partial P\left(\mathbf{r},\mathbf{r}_0,\omega\right)}{\partial \mathbf{n}}\right] \quad (10)$$

where $\mathbf{r}_b$ represents the location below M.S.(Figure 1)
Substitute (8) and (9) into above equation, and we can arrive at

$$A(\omega)\approx\frac{\int d\tilde{x}\left[\underbrace{P\left(\mathbf{r},\mathbf{r}_0,\omega\right)\frac{\partial G_0^D\left(\mathbf{r}_b,\mathbf{r},\omega\right)}{\partial \tilde{z}}}_{A}-\underbrace{G_0^D\left(\mathbf{r}_b,\mathbf{r},\omega\right)\frac{\partial T\left(\tilde{x},z,\mathbf{r}_0,\omega\right)}{\partial z}}_{B}\right]}{-G_0^D\left(\mathbf{r}_b,\mathbf{r}_0,\omega\right)+\underbrace{\int d\tilde{x}G_0^D\left(\mathbf{r}_b,\mathbf{r},\omega\right)\mathbf{R}(\tilde{x},z,\mathbf{r}_0,\omega)}_{C}} \quad (11)$$

Where

$$R(\tilde{x},z,\mathbf{r}_0,\omega)=\frac{\partial}{\partial z}G_0^{DD}(\tilde{x},z,\mathbf{r}_0,\omega)$$

$$\frac{\partial}{\partial z}T\left(\tilde{x},z,\mathbf{r}_0,\omega\right)=\int dx'\left[P\left(x',z',\mathbf{r}_0,\omega\right)\frac{\partial^2 G_0^{DD}\left(\tilde{x},z,x',z',\omega\right)}{\partial z\partial z'}\right]$$

In order to avoid the unstable due to denominator close to zero, we always choose z above M.S. in equation (11).

The triangle relationship states that measured values of $P(\mathbf{r},\mathbf{r}_s,\omega)$ and $\frac{\partial}{\partial n}P(\mathbf{r},\mathbf{r}_s,\omega)$ along a cable and $A(\omega)$ satisfy the exact equation (10). One might think Equ (8), when $\mathbf{r}$ is evaluated on the cable, procides a second independent relationship that would allow $A(\omega)$ to be directly determined from $P(\mathbf{r},\mathbf{r}_s,\omega)$ along the cable. However, Weglein and Amundsen (2003) demonstrate that these are the same relationship. If you temporarily ignore this fact, and substitute equation (8) into equation (10) to 'eliminate'$\frac{\partial}{\partial n}P(\mathbf{r},\mathbf{r}_s,\omega)$, then when $\mathbf{r}$ approaches cable, the expression in the denominator of equation (11) will be zero. The inverse is 'unstable'. To avoid this instability in the inversion, what is being suggested here is that values above the cable for $\frac{\partial}{\partial n}P(\mathbf{r},\mathbf{r}_s,\omega)$ and $\frac{\partial}{\partial n}G_0^{DD}(\mathbf{r},\mathbf{r}_s,\omega)$ are substituted for those at the cable in the integral to avoid the singularity. This has the effect of avoiding a singular division by solving a nearby perturbed problem with the anticipation that this will lead to a stable approximate solution.

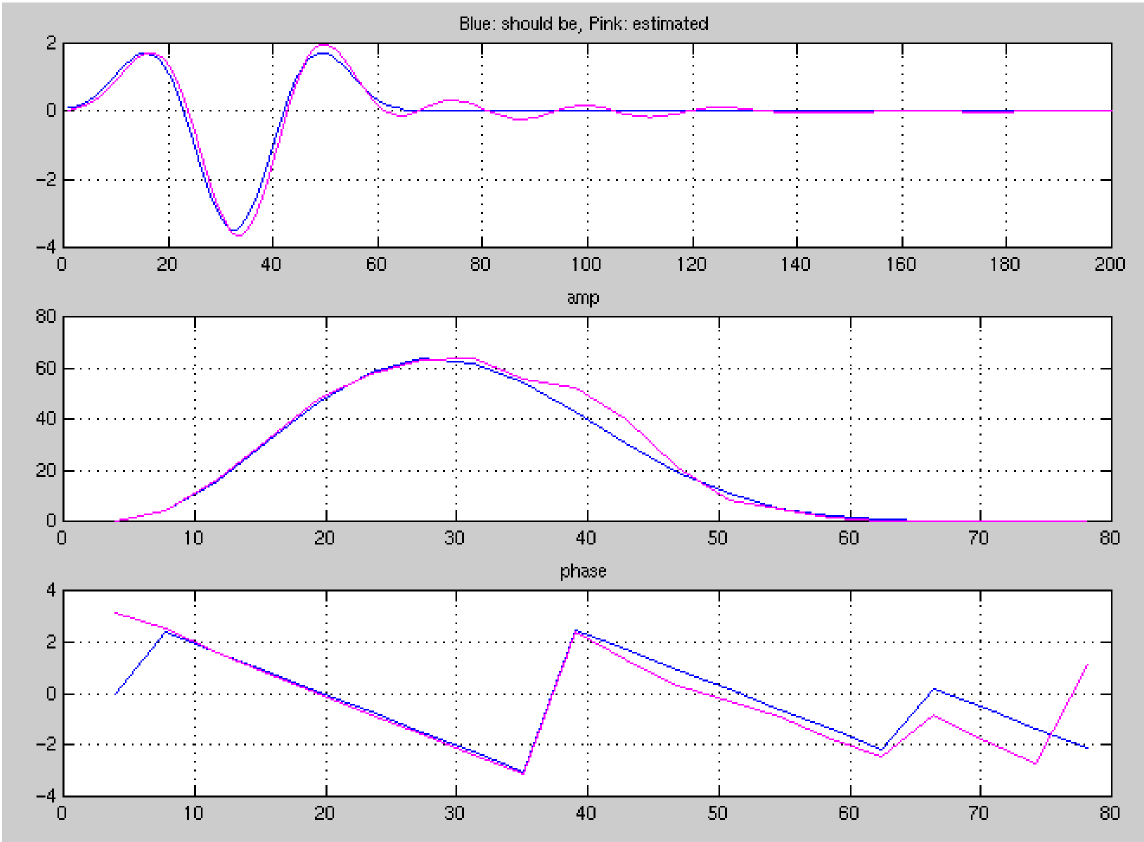

Figure 2: On top is the correct wavelet (blue) and final averaged wavelet (pink); in the middle are the amplitude spectra; and the bottom plot shows the phase spectra.

5 Synthetic example

We make three synthetic datasets with water depth 40m, 60m and 80m respectively. The source is 2m below the free surface, receivers are 6m below the free surface, the receiver interval is 2m (Figure 1). Then equation (10) was used to estimate the wavelet (figure 2). The wavelet estimated from water depth 80m reflector has least artifact; the final averaged of three estimated wavelets is close to the correct one.

Figure 3 shows the amplitudes and phases of correct wavelet and estimated one. We see there are some errors in amplitudes around 40 Hz, and phase change at 8 Hz and 75 Hz.

6 Conclusions

A method for estimating the wavelet directly from the data on a towed streamer was recently proposed by Weglein et al. (2002). That method proposed using the H. Tan (1999) wavefield prediction method to approximate the needed normal derivative along the cable. However, the wavelet method requires an integral over all receivers for a given shot, and the H. Tan prediction is not accurate under the source. In this paper, we propose addressing this problem by not dropping the term which is small only away from the source to achieve an algorithm that is valid for all offsets needed in the integral.

An intrinsic instability in this approach is addressed by seeking an approximate solution that replaces the unstable inversion by a "nearby" (i.e., perturbed) operation. Tests on synthetic data are encouraging; further tests are planned for noise stability and other issues.

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