A method for deghosting of towed streamer data without spectral division and without the need for an extra measurement

Notes

A.B. Weglein M-OSRP
S.A. Shaw, BP and M-OSRP
G.J.P. Correa, M-OSRP
Z. Guo, M-OSRP
T.H. Tan, Shell

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1 Summary

We present a method for removing receiver ghosts from towed streamer data. The method has the following properties

1. Only pressure measurements along a cable are required.

2. There is no spectral division.

3. The cable should consist of single sensor hydrophones.

4. When the source is above the cable, then the direct wave is also removed.

The method is derived using the Extinction Theorem. In the limit that we evaluate our result on the measurement surface, this theory corresponds to traditional up/down separation or P-Z summation. However, in principle, it produces the receiver deghosted data anywhere on *or above* the measurement surface.

This method makes use of the specific property of a Green's Function that becomes vanishingly small for typical towed streamer depths (6m) and seismic frequencies ($<$ 125 Hz).

2 Extinction Theorem for receiver deghosting

Green's Theorem states

$$\int_V (\Phi\nabla^2\Psi - \Psi\nabla^2\Phi)dV = \oint_S (\Phi\nabla\Psi - \Psi\nabla\Phi)\cdot\hat{n}\; dS \tag{1}$$

Consider a background medium that consists of a homogenous wholespace of water having velocity c_0. The Green's Function for this medium, G_0, satisfies

$$\left(\nabla^2 + \frac{\omega^2}{c_0^2}\right) G_0(\vec{r}\,|\vec{r}\,';\omega) = \delta(\vec{r} - \vec{r}\,') \tag{2}$$

In this derivation, we will assume that our actual medium is acoustic with constant density and variable velocity and therefore supports the wavefield P which satisfies

$$\left(\nabla^2 + \frac{\omega^2}{c^2(\vec{r})}\right) P(\vec{r}\,|\vec{r}_s;\omega) = A(\omega)\delta(\vec{r} - \vec{r}_s) \tag{3}$$

where $A(\omega)$ is the source signature.

We could also write an elastic wave equation for our actual medium.

Now define scattering (passive) sources as follows

$$\frac{1}{c^2(\vec{r})} = \frac{1}{c_0^2}\left[1 - \alpha_a(\vec{r}) - \alpha_e(\vec{r})\right] \tag{4}$$

where α_a and α_e describe the scattering potentials of the air (-water interface) and the earth, respectively (see Fig. 1). Then we can rewrite Eq.3 using Perturbation Theory

$$\begin{aligned}\left(\nabla^2 + k_0^2\right) & P(\vec{r}|\vec{r}_s;\omega) \\ & = A(\omega)\delta(\vec{r} - \vec{r}_s) + k_0^2\left[\alpha_a(\vec{r}) + \alpha_e(\vec{r})\right] P(\vec{r}|\vec{r}_s;\omega)\end{aligned} \tag{5}$$

where $k_0 = \frac{\omega}{c_0}$. Rearranging this equation gives

$$\begin{aligned}\nabla^2 P(\vec{r}|\vec{r}_s;\omega) = A(\omega)\delta(\vec{r} - \vec{r}_s) & - k_0^2 P(\vec{r}|\vec{r}_s;\omega) \\ & + k_0^2\left[\alpha_a(\vec{r}) + \alpha_e(\vec{r})\right] P(\vec{r}|\vec{r}_s;\omega)\end{aligned} \tag{6}$$

and from Eq.2 we have

$$\nabla^2 G_0(\vec{r}|\vec{r}\,';\omega) = -k_0^2 G_0(\vec{r}|\vec{r}\,';\omega) + \delta(\vec{r} - \vec{r}\,') \tag{7}$$

Substituting $\Phi = G_0(\vec{r}|\vec{r}\,';\omega)$ and $\Psi = P(\vec{r}\,'|\vec{r}_s;\omega)$ in Eq.1 we have

$$\begin{aligned}\int_{\vec{r}\,'\in V} & \left\{A(\omega)G_0(\vec{r}|\vec{r}\,';\omega)\delta(\vec{r}\,' - \vec{r}_s) + G_0(\vec{r}|\vec{r}\,';\omega)k_0^2\alpha_a(\vec{r}\,')P(\vec{r}\,'|\vec{r}_s;\omega)\right. \\ & \left. + G_0(\vec{r}|\vec{r}\,';\omega)k_0^2\alpha_e(\vec{r}\,')P(\vec{r}\,'|\vec{r}_s;\omega) - P(\vec{r}\,'|\vec{r}_s;\omega)\delta(\vec{r} - \vec{r}\,')\right\} dV \\ = & \oint_{\vec{r}\,'\in S} \left\{G_0(\vec{r}|\vec{r}\,';\omega)\nabla' P(\vec{r}\,'|\vec{r}_s;\omega) - P(\vec{r}\,'|\vec{r}_s;\omega)\nabla' G_0(\vec{r}|\vec{r}\,';\omega)\right\} \cdot \hat{n}\; dS\end{aligned} \tag{8}$$

Now consider the volume, V, bounded by the surface $S = S_0 + S_R$ depicted in Fig. 1. S_0 is the measurment surface, e.g., the towed streamer or ocean bottom cable. We notice that α_a is outside the volume and therefore does not contribute to the volume integral. The source location r_s is also outside the volume so $\delta(\vec{r}\,' - \vec{r}_s) = 0$ for all $\vec{r}\,'$ in V. We also choose the evaluation point r to be outside the volume by placing it above the measurement surface. Hence, $\delta(\vec{r} - \vec{r}\,') = 0$ for all $\vec{r}\,'$ in V. Then, if we make use of the causal Green's Function G_0^+ and apply the Sommerfeld radiation condition such that contributions from the surface S_R become zero as $R \to \infty$, Eq. 8 becomes

$$\int_{\vec{r}\,'\in V} G_0^+(\vec{r}|\vec{r}\,';\omega) k_0^2 \alpha_e(\vec{r}\,') P(\vec{r}\,'|\vec{r}_s;\omega) dV$$
$$= \int_{\vec{r}\,'\in S_0} \left\{ G_0^+(\vec{r}|\vec{r}\,';\omega) \nabla' P(\vec{r}\,'|\vec{r}_s;\omega) - P(\vec{r}\,'|\vec{r}_s;\omega) \nabla' G_0^+(\vec{r}|\vec{r}\,';\omega) \right\} \cdot \hat{n}\; dS_0 \tag{9}$$

The left-hand side of Eq. 9 is the receiver-deghosted scattered field, P_{rdg}. The volume integral contains no interactions with α_a; it can be interpreted as an infinite sum of propagations from the source and the free surface through the actual medium (P), scattering in the Earth ($k_0^2\alpha_e$), followed by propagation in water back to the receivers (G_0^+). P_{rdg} is only the upgoing portion of the total field. In addition, P_{rdg} also has the direct wave removed, meaning that it is the scattered field. This is evident because the direct wave has no interactions with the Earth and so is not represented by this integral. This property is important in shallow water areas, where the direct wave interferes with the reflection events and therefore is difficult to mute.

Equation 9 is a manifestation of the Extinction Theorem. We have extinguished the contribution to the total field that was due to the scattering sources above the measurement surface by choosing our output point ($\vec{r}$) in that region. In the limit that we evaluate P_{rdg} on the measurement surface, this theory describes conventional up-down separation, sometimes call P-Z summation. In the wavenumber domain, the surface integral is a weighted sum of the pressure measurements and the vertical component of particle velocity.

To calculate P_{rdg}, we evaluate the surface integral on the right-hand side of Eq. 9. This requires the measured total pressure data and its normal derivative. Assuming that we know the acoustic properties of water, we can straightfowardly calculate the Green's function and its normal derivative. Furthermore, if we were able to predict the total wavefield above the measurement surface, then we could calculate its normal derivative, rather than require its measurement.

3 Eliminating the need to measure the normal derivative

Tan (1999) points out that, for typical streamer depths ($\sim$ 6m) and seismic frequencies ($<$ 125 Hz), the following equation well-approximates the total wavefield above the cable

$$P(\vec{r}\,'|\vec{r}_s;\omega) \approx \int_{\vec{r}\,''\in S_0} \left\{P(\vec{r}\,''|\vec{r}_s;\omega)\nabla'' G_0^{DD}(\vec{r}\,'|\vec{r}\,'';\omega)\right\} \cdot \hat{n}\ dS_0 \tag{10}$$

where G_0^{DD} is a Green's function that vanishes (equals zero) at the free surface and on the measurement surface (Tan 1999, Osen et al. 1998). Weglein et al. (2000) propose that, since Eq. 10 gives us an infinite number of estimates of the total wavefield above the cable, we can predict the field at two or more depths and calculate its normal derivative. In 2-D, the vertical derivative can be expressed using Eq. 10 as

$$\frac{\partial}{\partial z'}P(x',z'|x_s,z_s;\omega) \approx \int_{-\infty}^{\infty} P(x'',z_c|x_s,z_s;\omega)\left[\frac{\partial^2}{\partial z'\partial z''}G_0^{DD}(x',z'|x'',z'';\omega)\right]_{\substack{z''=z_c\\ z=z_c-\epsilon}} dx'' \tag{11}$$

where z_c is the cable depth and ϵ is a small positive number. As $\epsilon \to 0^+$, Eq. 11 is the vertical derivative of the total pressure field on the cable. Rewriting Eq. 9 in 2-D and then substituting in Eq. 11, we have

$$\begin{aligned} P_{rdg}(x,z|x_s,z_s;\omega) \approx \int_{-\infty}^{\infty} &\Big\{G_0^+(x,z|x',z_c;\omega) \\ &\int_{-\infty}^{\infty} P(x'',z_c|x_s,z_s;\omega)\left[\frac{\partial^2}{\partial z'\partial z''}G_0^{DD}(x',z'|x'',z'';\omega)\right]_{\substack{z''=z_c\\ z=z_c-\epsilon}} dx'' \\ &-P(x',z_c|x_s,z_s;\omega)\frac{\partial}{\partial z'}G_0^+(x,z|x',z';\omega)\Big\}\, dx' \end{aligned} \tag{12}$$

This is an expression for the upgoing wavefield in terms of only pressure measurements on the cable, and analytic reference Green's Functions.

We propose to code the algorithm for P_{rdg} (Eq. 12) and test its effectiveness by first applying it to synthetic data. For example, Figs. 2 and 3 have been prepared for this purpose. We can clearly see the effect that the free surface has on the seafloor reflector by comparing these two shot records. When we have completed this test, we will test the algorithm on field data. Experience has shown that these methods require a good recording of the direct wave. This is achievable with single sensor data. The typical recording arrays are designed to dampen the direct wave and to emphasize the upgoing waves. This proved to be an impediment to implementing previous wavelet estimation methods (Weglein and Secrest, 1990) that were also derived using Extinction Theorem.

4 Discussion

The sensitivity in traditional deghosting arises from using the boundary condition $P(z'=0)$ and the measurement $P(z'=z_c)$. Using $P(z'=z_c)$ and $\frac{\partial P}{\partial z'}(z'=z_c)$ avoids the spectral division. The Extinction Theorem, and the confluence of depths of cable and $f < 125$ Hz, combine to allow an algorithm that requires only measurements $P(z'=z_c)$ to receiver deghost the data.

It is instructive to ask whether we can treat the free-surface as a source and use the Extintion Theorem to remove all its effects, i.e., ghosts *and* free surface multiples. The answer to this question is no for multiples, but yes for receiver ghosts. The Extinction Theorem will eliminate all events whose last interaction was on the side of the measurement surface that you choose to evaluate the wavefield. Free surface multiples have more complicated interactions with scattering sources α_a and α_e.

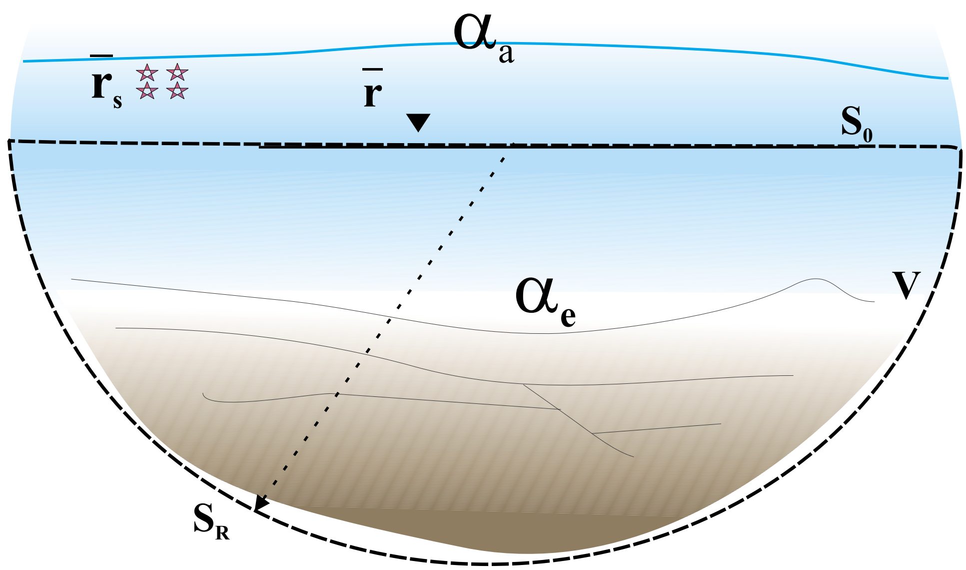

Figure 1: Description of seismic experiment. Two passive scattering sources are superimposed on a homogenous wholespace of water. The measurement surface is the cable of receivers. The active source is the air gun array and the two passive sources are the Earth scattering below the measurement surface, α_e, and the air at and above the free surface α_a.

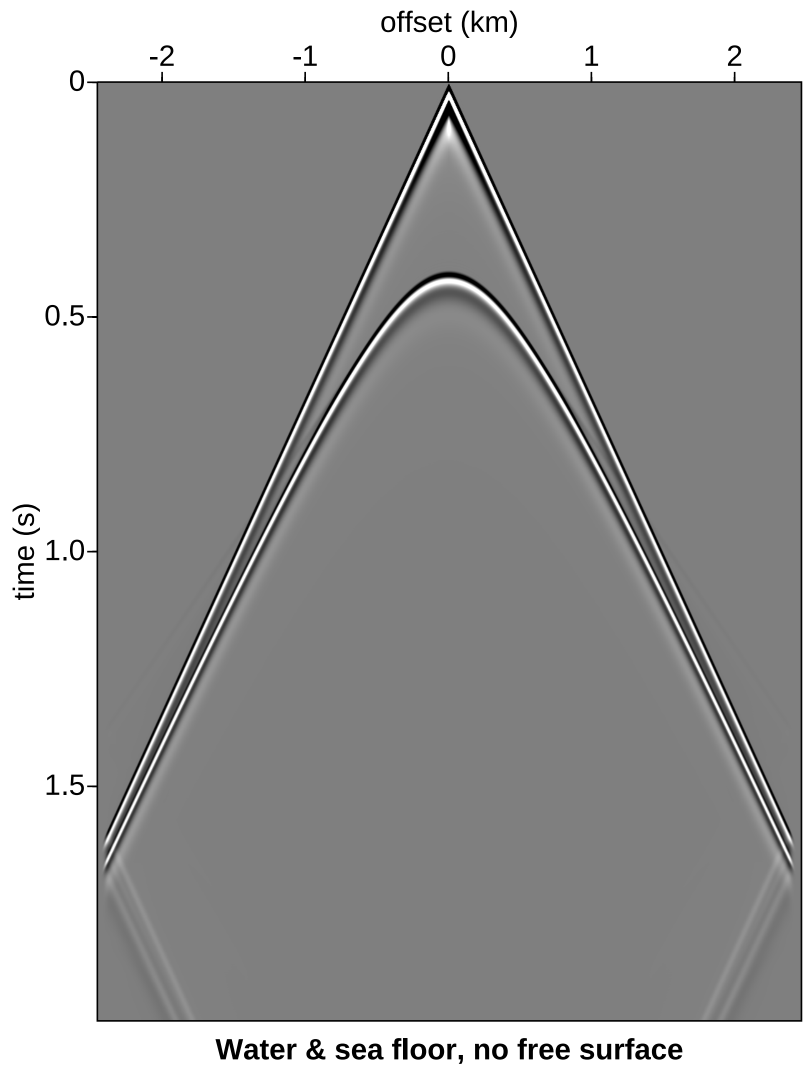

Figure 2: A synthetic shot record for a single reflector in the absence of a free surface. There are no ghosts and no multiple reflections.

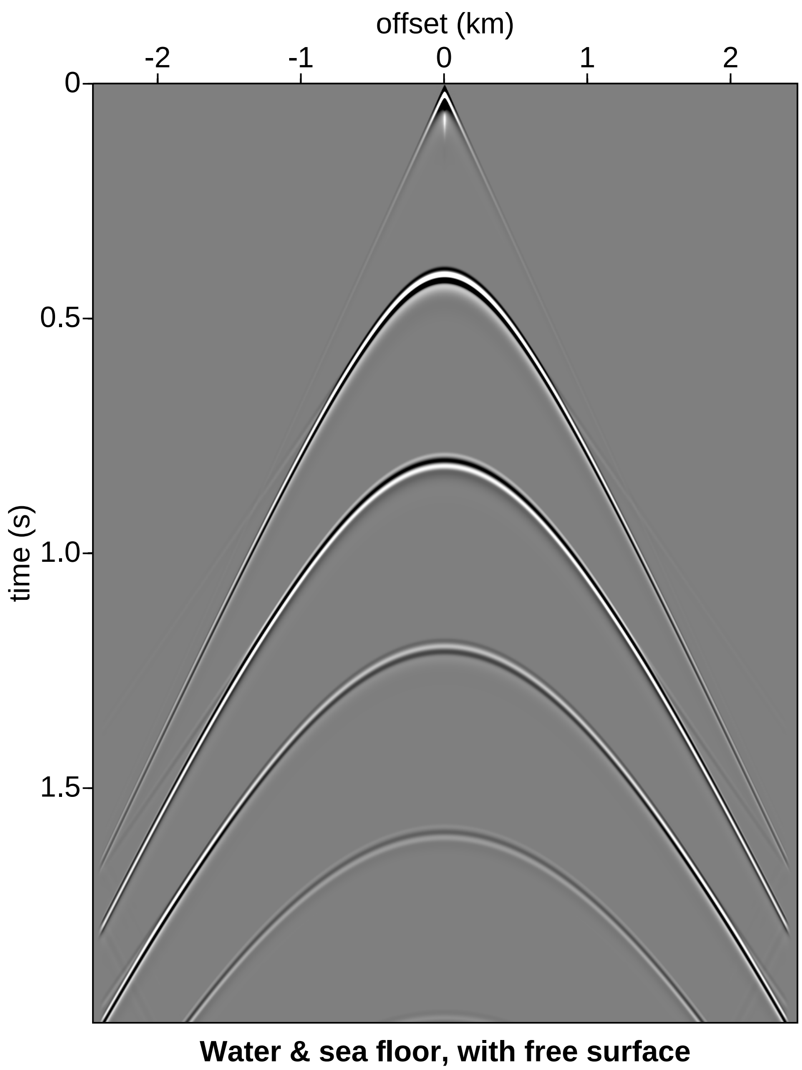

Figure 3: A synthetic shot record for a single reflector in the presence of a free surface. The effect of the ghosts and multiple reflections are evident when compared with Fig. 2.