Analysis of the Inverse Scattering Series (ISS) internal multiple attenuation and elimination algorithms as effective tool box choices for absorptive and dispersive media with interfering events

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SUMMARY

The Inverse Scattering Series (ISS) internal multiple attenuation algorithm can predict the exact time and approximate amplitude of every internal multiple at all offsets at once. This algorithm does not require any subsurface information and it is model-type independent. When the primaries and multiples are isolated, the ISS internal multiple attenuation algorithm plus energy minimization adaptive subtraction can effectively eliminate internal multiples independent of the the medium model type (e.g. acoustic, elastic, anisotropic, inelastic, etc.). However, when internal multiples are proximal to and/or interfering with a primary, the energy-minimization adaptive subtraction can fail. In proximal /interfering cases the ISS elimination algorithm is needed for predicting the exact time and exact amplitude of multiples, and it would not depend on the energy minimization criteria to fill the gap between attenuating and eliminating the internal multiple. Thus M-OSRP proposed developing ISS internal multiple elimination algorithm to accommodate these proximal/interfering cases. We have an interest in examining the issue of the elimination of interfering internal multiples for increasingly realistic subsurface circumstances. We also recognize the benefit of studying each step of added realism and complexity in isolation. Absorption/dispersion can have a very significant impact on amplitude, often more significant than the acoustic/elastic differences. There is a line of research in the ISS initiative that extends the development and analysis to the absorptive/dispersive world by studying an acoustic absorptive medium. For example, Innanen and Weglein (2003, 2005); Innanen and Lira (2008, 2010); Wu and Weglein (2014). This paper follows that line of contributions and extends ISS internal multiple elimination to absorptive/dispersive acoustic medium, which is the simplest world with an absorptive/dispersive property. We test the current ISS internal multiple elimination algorithm on synthetic data (P-only events) from attenuating medium both analytically and numerically. The analysis and results of the tests show that the current elimination algorithm predicts P-only internal multiple in an absorptive/dispersive medium with both the exact time and amplitude if the absorption/dispersion (finite Q) is only located beneath the generator (which is where the the downward reflection occurs) of the first-order internal multiple, without knowing the medium and its absorptive/dispersive properties. Under this type of circumstances the current ISS internal multiple elimination algorithm is fully effective in predicting accurate P-only internal multiples in an absorptive/dispersive medium. That is positive news for the exploration plays where absorption is only significant below the major internal multiple generators. For instance, that can be the situation for a single absorptive salt body. In this case the major internal multiple generator is often either the water bottom or the top of the salt body and the major attenuation happens within the salt body. Thus the absorption is only existing below the generator, the current acoustic based elimination algorithm is sufficient for predicting an effective P-only internal multiple in this type of exploration play.

INTRODUCTION

Internal multiple removal is a long-standing problem in exploration seismology. Many methods have been developed including: stacking, FK filter, Radon transform, deconvolution and Feedback loop. These methods make either statistical assumptions, assume move-out differences, or require knowledge of the subsurface and the generators of the multiples. As a direct response, the ISS internal multiple algorithms (both of attenuation and elimination algorithms), made none of these limiting assumptions. Among them, the ISS internal multiple attenuation algorithm (Araújo et al., 1994; Weglein et al., 1997, 2003) predicts the exact time and approximate amplitude of every internal multiple at all offsets. In addition to not requiring any subsurface information, the ISS internal multiple attenuation algorithm is model-type independent, that is, they are exactly the same unchanged algorithm for an acoustic, elastic, isotropic, anisotropic and inelastic subsurface. When combined with an energy minimization adaptive subtraction, the ISS internal-multiple attenuation algorithm can effectively eliminate internal multiples when the primaries and internal multiples are isolated. For example, Matson (1999) and Fu et al. (2010) are the first cases of ISS internal multiple attenuation algorithm on marine and land field data, respectively. And there are numerous following cases (Luo et al., 2011; Ferreira, 2011; Ferreira et al., 2013; de Melo et al., 2013; Hung et al., 2014; Fu and Weglein, 2014) from both major services companies and petroleum companies. However, when internal multiples are proximal to and/or interfering with a primary, the criteria of energy-minimization adaptive subtraction can fail (e.g., the energy can increase when a multiple is removed from a event containing a destructively interfering primary and multiple). In proximal/interfering cases ISS elimination algorithm is needed for predicting the exact time and exact amplitude of multiples, and it would not depend on the energy minimization criteria to fill the gap between attenuating and eliminating the internal multiples. The initial idea to achieve an elimination algorithm is developed by Weglein and Matson (1998) by removing attenuation factors (the difference between the predicted internal multiples and true internal multiples) using reflection data. There are also early discussions in Ramírez (2007). Based on the ISS attenuation algorithm and the initial idea for elimination, Herrera and Weglein (2013) formulated an ISS algorithm for a normal incident wave on a 1D earth, that eliminate first-order internal multiples only generated by the shallowest reflector. Zou and Weglein (2013) then advanced and extended these initial contributions for all first order internal multiples generated at all reflectors. Zou and Weglein (2015) generalized the ISS elimination algorithm into 1D pre-stack medium. Zou and Weglein (2016) further provided the first multi-dimensional ISS elimination algorithm for all first order internal multiples at all offsets.

We have an interest in examining the issue of elimination interfering internal multiples for increasingly realistic subsurface circumstances. We also recognize the benefit of studying each step of added realism and complexity in isolation. Absorption/dispersion can have a very significant impact on amplitude, often more significant than the acoustic/elastic differences. There is a line of research in the ISS history and initiative that extends to the absorptive/dispersive world by studying an acoustic absorptive medium. For example, Innanen and Weglein (2003, 2005); Innanen and Lira (2008, 2010); Wu and Weglein (2014). This paper follows that line of contributions to extend ISS internal multiple elimination to an absorptive/dispersive acoustic medium, which is the simplest world with an absorptive/dispersive property.

ANALYTICAL TEST AND ANALYSIS

Wavefield Expression For an Attenuating Medium

Based on Aki and Richards (2002), assuming a constant Q model, the 1D wave equation can be written as

$$\frac{d^2P}{dz^2} + \frac{\omega^2}{c_0^2}\left(1+\frac{F(\omega)}{Q}\right)P = 0, \tag{1}$$

where

$$F(\omega) = \frac{i}{2}sgn(\omega) - \frac{1}{\pi}log\left(\left|\frac{\omega}{\omega_r}\right|\right); \tag{2}$$

and the right-hand side has two terms: the first term is related to the energy attenuation, and the second term is related to velocity dispersion. ω_r here is the reference frequency, and it could be chosen as the maximum frequency in the experiment; c_0 is the constant velocity at the reference frequency. Q here is used to represent the energy loss for a wave-field propagating, in one wave length, and is defined as

$$Q = \frac{2\pi E}{\Delta E}, \tag{3}$$

where E is the energy of the wave-field, and ΔE is the energy loss in a wavelength of propagation. If we define a frequency dependent velocity $c(\omega)$ as

$$\frac{1}{c(\omega)} = \frac{1}{c_0}\left(1+\frac{F(\omega)}{Q}\right), \tag{4}$$

then wavefield $P(z,\omega)$ can be expressed as a one-way wave propagating solution to equation 1:

$$P(z,\omega) = e^{i\frac{\omega}{c(\omega)}z} = e^{i\frac{\omega}{c_0}z}e^{-\frac{\omega sgn(\omega)}{2c_0Q}z}e^{-i\frac{\omega}{c_0Q}log\left(\left|\frac{\omega}{\omega_r}\right|\right)z}. \tag{5}$$

From the formula, we understand the wavefield is influenced by three terms: the first term is contributing to the phase with the velocity of c_0, the second term is contributing to the energy attenuation, and the third term is contributing to the phase delay with velocity dispersion. Only the first term remains when Q is increased to infinity; i.e, the medium is back to no absorption/dispersion.

Analytical Test in the 1D Normal Incidence Case

Following the explanation in the previous section, we can express the wave-field in an absorptive/dispersive medium analytically. In this section, the absorptive /dispersive data will be used as input to test the ISS internal multiple elimination algorithm analytically.

For 1D normal incidence, the ISS internal multiple elimination algorithm $b_E(k_z)$ defined in Zou and Weglein (2015) as:

$$b_E(k_z) = \int_{-\infty}^{\infty} dz e^{ik_z z} b_1(z) \int_{-\infty}^{z-\varepsilon_1} dz' e^{-ik_z z'} F[b_1(z')] \times \int_{z'+\varepsilon_2}^{\infty} dz'' e^{ik_z z''} b_1(z''), \tag{6}$$

where $b_1(z)$ is the water speed uncollapsed Stolt migration of the data; $b_E(k_z)$ is the elimination algorithm internal multiple prediction in wavenumber domain; $F[b_1(z)]$ and $g(z)$ are two intermediate functions defined as following:

$$F[b_1(z)] = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dz' dk_z'$$
$$\times \frac{e^{-ik_z'z}e^{ik_z'z'}b_1(z')}{[1-\int_{-\infty}^{z'-\varepsilon}dz''b_1(z'')e^{ik_z'z''}\int_{z''-\varepsilon}^{z''+\varepsilon}dz'''g(z''')e^{-ik_z'z'''}]^2}$$
$$\times \frac{1}{1-|\int_{z'-\varepsilon}^{z'+\varepsilon}dz''g(z'')e^{ik_z'z''}|^2}; \tag{7}$$

$$g(z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dz' dk_z'$$
$$\times \frac{e^{-ik_z'z}e^{ik_z'z'}b_1(z')}{1-\int_{-\infty}^{z'-\varepsilon}dz''b_1(z'')e^{ik_z'z''}\int_{z''-\varepsilon}^{z''+\varepsilon}dz'''g(z''')e^{-ik_z'z'''}}. \tag{8}$$

A two-reflector 1D model is used in this analytical calculation, and with the depths of source and receiver both assumed to be zero.

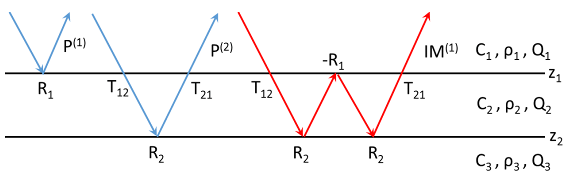

Figure 1: A two-reflector 1D model. $P^{(1)}$ and $P^{(2)}$ are primaries from the first and the second interface, respectively; $IM^{(1)}$ is the first order internal multiple; R_1 and R_2 are reflection coefficients; T_{12} and T_{21} are transmission coefficients; c_1, c_2 and c_3 are the velocities; ρ_1, ρ_2 and ρ_3 are densities; and Q_1, Q_2 and Q_3 are quality factors.

For a 1D model and a 1D normal-incident plane wave, two primaries in the data $D(\omega)$ can be represented as:

$$P^{(1)}(\omega) = R_1(\omega)e^{i\frac{\omega}{c_1(\omega)}2z_1}, \tag{9}$$

$$P^{(2)}(\omega) = T_{12}(\omega)T_{21}(\omega)R_2(\omega)e^{i\frac{\omega}{c_1(\omega)}2z_1}e^{i\frac{\omega}{c_2(\omega)}2(z_2-z_1)}, \tag{10}$$

where $\frac{1}{c_1(\omega)} = \frac{1}{(c_1)}\left(1+\frac{F(\omega)}{Q_1}\right)$ and $\frac{1}{c_2(\omega)} = \frac{1}{(c_2)}\left(1+\frac{F(\omega)}{Q_2}\right)$ and $F(\omega)$ is defined in equation 2. The two primaries are both suffering from the absorption and dispersion.

After migrating the data into the pseudo depth domain to get $b_1(z)$, we can substitute it into ISS internal multiple elimination algorithm (equation 6-8). We further assume that the two primaries are isolated to make sure there is no overlap between the two events among the integrals. And we use the same technique in Herrera and Weglein (2013) to simplify the calculation since in this case the internal multiple is only generated by the shallowest reflector. The predicted first-order internal multiple $b_E(k_z)$ can be obtained:

$$b_E(k_z) = \frac{(T_{12}(\omega)T_{21}(\omega))^2}{1-\left(R_1(\omega)e^{-\frac{|k_z|}{Q_1}z_1}\right)^2}R_1(\omega)R_2^2(\omega)e^{-\frac{|k_z|}{Q_1}z_1}e^{i\frac{\omega}{c_1(\omega)}2z_1}e^{i\frac{\omega}{c_2(\omega)}2(z_2-z_1)}. \tag{11}$$

Analysis on the analytical test

We first assume the absorption/dispersion happens both above and below the generator of internal multiple (and later we will see the circumstance when the absorption /dispersion happens only below the generator). Under this assumption, the actual first-order internal multiple can be calculated in the wavenumber domain analytically:

$$IM(k_z) = -T_{12}(\omega)T_{21}(\omega)R_1(\omega)R_2^2(\omega)e^{i\frac{\omega}{c_1(\omega)}2z_1}e^{i\frac{\omega}{c_2(\omega)}2(z_2-z_1)}. \tag{12}$$

The relation between the predicted and the actual first-order internal multiple is

$$b_E(k_z) = -\frac{T_{12}(\omega)T_{21}(\omega)}{1-\left(R_1(\omega)e^{-\frac{|k_z|}{Q_1}z_1}\right)^2}e^{-\frac{|k_z|}{Q_1}z_1}IM(k_z). \tag{13}$$

In equation 13, it can be seen that the predicted amplitude is not exactly accurate for input data with Q absorption, however, we still get the correct phase. Also we can find only the Q above the generator (Q_1) appears in the relation between the predicted and the actual first-order internal multiple; where as the Q below generator (Q_2 and Q_3) are not affect the predicted multiple.

Given the the relation between transmission and reflection coefficients for P wave,

$$T_{12}(\omega)T_{21}(\omega) = 1 - R_1(\omega)^2, \tag{14}$$

if above the generator the medium do not have Q absorption ($Q_1 \to \infty$) and no matter what the Q values below the generator (Q_2 and Q_3) are, we can always obtain accurate actual internal multiple (with the opposite polarity) for P-only events.

$$b_E(k_z) = -\frac{T_{12}(\omega)T_{21}(\omega)}{1-(R_1(\omega))^2}IM(k_z) = -IM(k_z),$$
$$\text{when } Q_1 \to \infty. \tag{15}$$

NUMERICAL TESTS

In this section, we preform two numerical tests to confirm the conclusion we drew in the previous section. In each test, we first numerically generate the synthetic data

	Layer Number	Velocity (m/s)	Density (kg/m^3)	Q Value
Model 1	1	1500	1000	60
	2	6500	1000	40
	3	2000	1000	60
Model 2	1	1500	1000	60000
	2	6500	1000	40
	3	2000	1000	60

Table 1: The parameters of two two-reflector 1D models.

in frequency-wavenumber domain, then we use ISS internal multiple elimination algorithm to calculate the predicted first-order internal multiple event, and finally we compare the predicted internal multiple with the internal multiple in the synthetic data. Two two-reflector 1D models (like in figure 1) will be used in the two tests. The parameters of the two models are listed in Table 1. The first model represents the Q existing in every layer, where as in the second model Q only is significant below the generator. The comparisons between real multiple (blue solid line) and predicted multiple (red dot line) for both models are shown in Fig 2. We can see if the absorption /dispersion is only significant beneath the multiple generator (Figure 2b), the acoustic ISS internal multiple elimination algorithm is fully effective in predict accurate P-only internal multiples, which is consistent with the conclusion we drew in the previous section.

CONCLUSIONS

The analysis and results of the tests show that the current (acoustic based) elimination algorithm predicts P-only internal multiple in an absorptive/dispersive medium with both the exact time and amplitude if the absorption /dispersion (finite Q) is only located beneath the generator of the internal multiple, without knowing the medium and its absorptive/dispersive properties. Under this type of circumstance the current ISS internal multiple elimination algorithm is fully effective predicting accurate P-only internal multiples. That is positive news for the exploration plays where absorption is only significant below the major internal multiple generators. For instance, one of many cases for such a circumstance would be a single absorptive salt body. In this case the major internal multiple generator is either the water bottom or the top of the salt body and the major attenuation happens within the salt body. Thus in the circumstance that absorption is only existing below the generator, the current acoustic based elimination algorithm is sufficient for predicting an effective P-only internal multiples in absorptive/dispersive media.

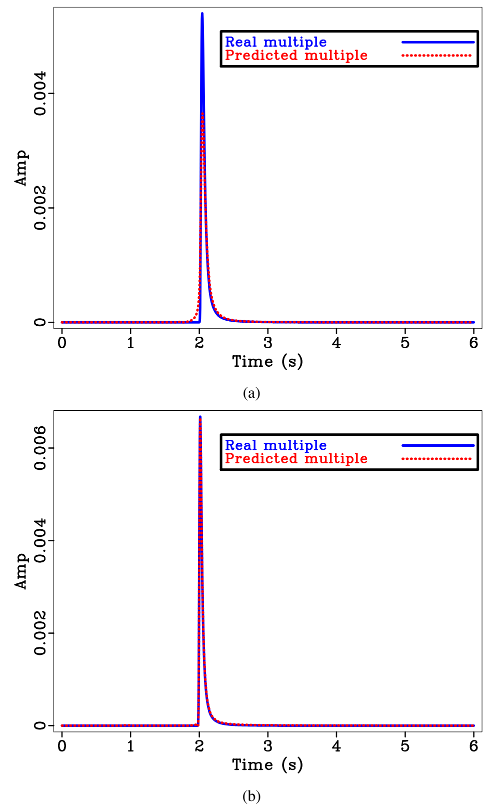

Figure 2: The comparison between real multiple (blue solid line) and predicted multiple (red dot line): (a) for model 1, in which the absorption/dispersion is significant both above and beneath the multiple generator; (b) for model 2, in which the absorption/dispersion is only significant beneath the multiple generator. If the absorption/dispersion is only significant beneath the multiple generator (like in model 2), the acoustic ISS internal multiple elimination algorithm is fully effective in predicting accurate P-only internal multiple.

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