Analysis, testing and comparison of the Inverse Scattering Series (ISS) Free-Surface Multiple-Elimination (FSME) algorithm, and the industry-standard SRME plus energy minimization adaptive subtraction

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SUMMARY

This abstract compares the ISS FSME and SRME plus-energy-minimization adaptive subtraction for removing both isolated free-surface multiples and those free-surface multiples that interfere with primaries, without subsurface information and without damaging the primaries. We provide a guide to when each is the appropriate choice within the free-surface-multiple-removal toolbox. It will be shown that SRME plus energy-minimization adaptive subtraction can be effective for isolated free-surface multiples that are not proximal to other events. The ISS FSME algorithm is the appropriate choice when the free-surface multiple is proximal to or interfering with other events.

INTRODUCTION

Multiple removal is a longstanding problem in exploration seismology. Although methods for removing multiples have advanced and have become more effective, the concomitant industry trend toward more complex exploration areas and difficult plays has at times outpaced advances in multiple-attenuation capability. The topic of multiples, and the need for developing ever more effective methods for their removal, remains high in terms of industry interest, priority and research investment. We advocate a tool-box approach and seek to understand the place and role that each method within the toolbox plays within the spectrum of different capabilities and responses, and how to choose the method that's a best match for the user's application and objective.

The cost-effective and appropriate choice depends on the complexity of the geology, the data, and processing objective. If one can well estimate the velocity of primaries and there is sufficient moveout between primaries and multiples then Radon methods (Foster and Mosher, 1992; Trad et al., 2003; Xue et al., 2016) are often the indicated choice. If the free-surface multiples are isolated (and temporally distinct from primaries) the SRME (Berkhout, 1985; Verschuur et al., 1992), which predict approximate amplitude and time, followed by adaptive subtraction is an effective strategy. The Inverse Scattering Series (ISS) for free-surface multiples (Carvalho et al., 1991; Weglein et al., 1997), in principle, predicts the amplitude and phase of free-surface multiples at all offsets, and doesn't require an adaptive subtraction and can eliminate the multiple in the presence of proximal or interfering events.

The ISS method is more costly than Radon and SRME followed by adaptive subtraction, but can be the cost effective choice when the goal is the surgical removal of free surface multiples that are proximal to primaries or other multiples of different orders and without damaging the primary.

THE ISS FSME

Carvalho et al. (1991) and Weglein et al. (1997, 2003) developed the ISS FSME algorithm from the Inverse Scattering Subseries for removing free-surface multiples (See Equ. 1 and 2).

$$D'(k_g,k_s,\omega) = \sum_{n=1}^{\infty} D'_n(k_g,k_s,\omega), \tag{1}$$

where D'_n is calculated as follows,

$$D'_n(k_g,k_s,\omega) = \frac{1}{i\pi A(\omega)} \int dk\, q\, e^{iq(\varepsilon_g+\varepsilon_s)} D'_1(k_g,k,\omega) D'_{n-1}(k,k_s,\omega),$$
$$n = 2,3,4,.... \tag{2}$$

The first term in this algorithm is the input data, $D'_1(k_g,k_s,\omega)$, in a 2D case, which is the Fourier transform of the deghosted prestack data, and with the direct wave removed. The subsequent prediction terms, represented by $D'_2, D'_3, ...$, provide predictions of free-surface multiples of different orders. Specifically, each term in D'_n (where $n = 2,3,4...$) performs two functions: (1) it eliminates the nth order free-surface multiple and (2) it alters all higher order free-surface multiples to be prepared to be removed by higher-order D'_{n+1} term.

The sum of these predictions $(D'_2 + D'_3 + ... + D'_{n+1})$ will provide free-surface-multiple predictions with accurate time and accurate amplitude (in opposite polarity) for free-surface multiples up to n-th order (Weglein et al., 2003; Zhang and Shaw, 2010; Ma and Weglein, 2016).

The data with free-surface multiples eliminated, D', is obtained by Equ.1.

$A(\omega)$, ε_g and ε_s in Equ. 2 are source signature, receiver depth and source depth, respectively; $q = \sqrt{\frac{\omega^2}{c_0} - k^2}$.

Assuming the removal of the direct wave, and the removal of source and receiver ghosts, the multiple prediction in SRME algorithm (Berkhout, 1985; Verschuur et al., 1992) can be expressed as follows

$$M(x_g,x_s;\omega) = \int P(x_g,x;\omega) P(x,x_s;\omega) dx. \tag{3}$$

The input, P, is the prestack data for one frequency component. Notice that, the input P for SRME and input D'_1 for ISS FSME are same and both assume the removal of direct wave and source and receiver ghosts.

The output, M, is the predicted free-surface-multiple model. This predicted free-surface multiple model is then subtracted adaptively from the data to obtain the data without free-surface multiple.

As pointed out in Weglein et al. (1997), the convolutional model of SRME misses the obliquity factor (q), compared with the

ISS free-surface multiple prediction. Hence, the SRME predicts the free-surface multiples with approximate amplitude and approximate time and requires an adaptive subtraction to remove free-surface multiples. In the next section, we will examine the difference between ISS FSME and SRME + adaptive using numerical examples.

NUMERICAL TESTS OF ISS FSME AND SRME

In this section, we numerically examine the prediction results from the ISS FSME (i.e., Equ. 2 where $n = 2$) and SRME prediction results (i.e., Equ. 3) and compare the multiple prediction results with the actual multiple in the data. We also examine the results after ISS FSME and SRME + adaptive subtraction, and compare the results with the actual primary in the data.

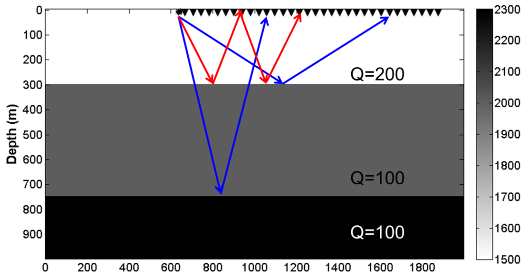

Figure 1: A 1D subsurface model with two primary events and one free-surface multiple event.

Figure 1 shows the model we used to generated input data using reflectivity method.

Notice that, in our example, (1) only three events (two primaries and one free-surface multiple) are generated, (2) the depths of the reflectors and velocities are chosen such that the second primary destructively interferes with the free-surface multiple, (3) we examine two cases where the data is generated with and without absorption in the model, see Table 1. Notice that, the only difference between these two tests are the input data, input data for Test 1 are generated without absorption in the model and input data for Test 2 are generated with absorption in the model.

Without absorption	With absorption
Test 1	Test 2

Table 1: We generate two input data (one with Q absorption, the other without Q absorption).

Test 1: Without absorption

For the first test has input data generated by the 1D subsurface without any absorption, Figure 2 shows the synthetic input data (a), multiple prediction result from ISS FSME (b) and SRME (c), actual primary (d), and result after ISS FSME (e) and SRME (f). Notice the result from ISS FSME was obtained by directly adding the ISS prediction result to the data without adaptive procedure, whereas the result from SRME was obtained by combining the prediction result from SRME and adaptive procedure.

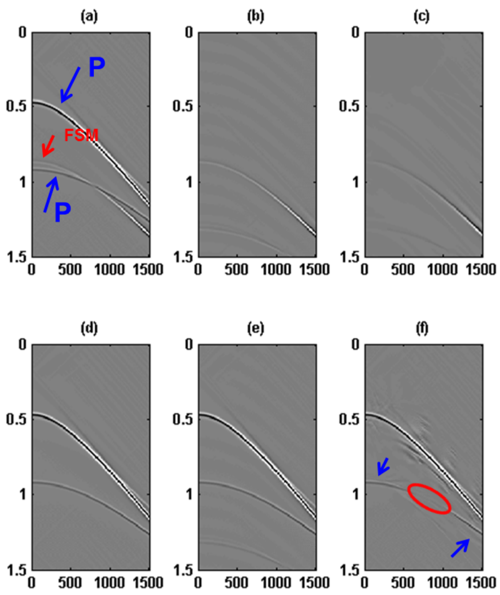

Figure 2: Result for the test which has the input data generated without absorption in the model. Input data (a), multiple prediction result from ISS FSME (b) and SRME (c), actual primary (d), and result after ISS FSME (e) and SRME + Adaptive (f).

Comparing the actual primary (Figure 2(d)) with the result after ISS FSME (Figure 2(e)), we find that, with the accurate multiple prediction, the ISS FSME can surgically remove the free-surface multiple and recover the primary.

Comparing the data (Figure 2 (a)) with the result after SRME + adaptive (Figure 2 (f)), we notice, combining the approximate multiple prediction with the adaptive subtraction, the SRME can remove isolated multiple successfully. The isolated free-surface multiple in Figure 2 (a) is removed in Figure 2 (f). In Figure 2 (f) the arrows point to the removed free-surface multiple. But the adaptive procedure can easily damage the primary which interferes with the multiple (red circle in Fig. 2 (f)).

Figure 3 provides trace plots for this test to examine the results in detail. The top five traces in Figure 3 show the comparison between the data (Black line) and two prediction results from ISS (Red line) and SRME (Green line) at different offsets. From the offsets 100m, 500m, 1000m and 1250m, where primary and multiple do not overlap, we can clearly see the ISS multiple prediction matches the actual multiple in the data, whereas the SRME prediction shows a disagreement.

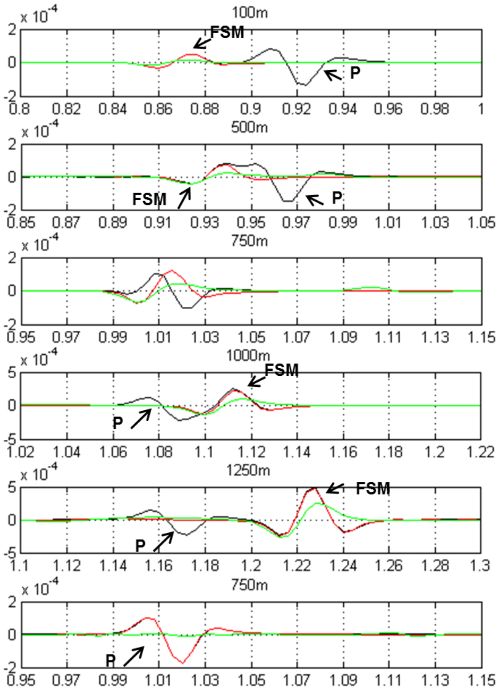

Figure 3: Result for the test which has the input data generated without absorption in the model. The top five traces show a trace comparison between the data and two predictions at different offsets. Black, red and green lines represent actual data (consisting of primary and free-surface multiple), ISS FSME prediction and SRME prediction, respectively. At offset 750m, the bottom trace shows the comparison between the actual primary (black) in the data and result after ISS (red) and SRME+adaptive (green) algorithm. The SRME + Adaptive damages the primary that interferes with the free surface multiple. The ISS free-surface algorithm effectively removes the free surface multiple without damaging the primary.

Notice that, at offset 750m, the primary and multiple overlap. The bottom trace shows the comparison between the actual primary (Black line) with the multiple-removal result after ISS FSME (Red line) and the multiple-removal result after SRME+adaptive (Green line) at offset 750m. This last trace shows the primary can be recovered by ISS FSME whereas the SRME combined with the adaptive could damage the primary.

Test 2: With absorption

Weglein et al. (2003) showed the model-type independent properties of both ISS free-surface multiple elimination algorithm and internal multiple attenuation algorithm. The meaning of model-type independent is that the the removal of free-surface multiples is achievable with precisely the same algorithm for an entire class of earth model types. The members of the model type class include acoustic, elastic and certain anelastic media.

Here, we provide a numerical example to demonstrate and confirm the effectiveness of the ISS FSME algorithm for an absorptive media.

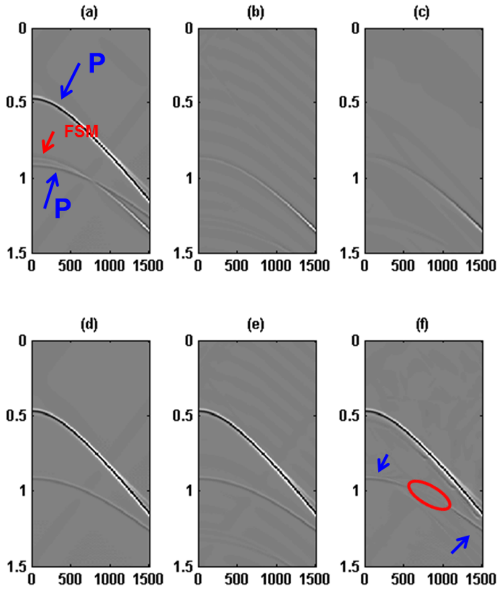

Figure 4: Result for the test which has the input data generated with absorption in the model. Input data (a), multiple prediction result from ISS FSME (b) and SRME (c), actual primary (d), and result after ISS FSME (e) and SRME + Adaptive (f).

Figures 4 (a), (b) and (c) show the shot gather comparison between the input data, ISS and SRME free-surface multiple prediction, respectively. Figure 4 (d), (e) and (f) show the shot gather comparison between the actual primary in the data, result after ISS FSME and result after SRME + adaptive, respectively. The comparisons show the ISS FSME can surgically remove the free-surface multiple and recover the primary. The SRME + adaptive can remove isolated multiple successfully, but the adaptive procedure can easily damage the primary which interferes with the multiple.

The top five traces in Figure 5 show the comparison between the data (Black line) and two prediction results from ISS (Red line) and SRME (Green line) at offset 100m, 500m, 750m, 1000m and 1250m, respectively. The bottom trace shows the comparison between the actual primary (Black line) with the multiple-removal result after ISS FSME (Red line) and the multiple-removal result after SRME+adaptive (Green line) at offset 750m. This last trace shows the primary can be recovered by ISS FSME whereas the SRME combined with the adaptive could damage the primary.

Examining the result in this test, we can conclude, for data generated by an acoustic media that's absorptive, the same ISS FSME algorithm remains effective to accurately predict the free-surface multiple and can surgically remove free-surface multiples that interfere with primaries, without damaging primaries. Thereby, we numerically confirm the model-type independent property of the ISS FSME algorithm.

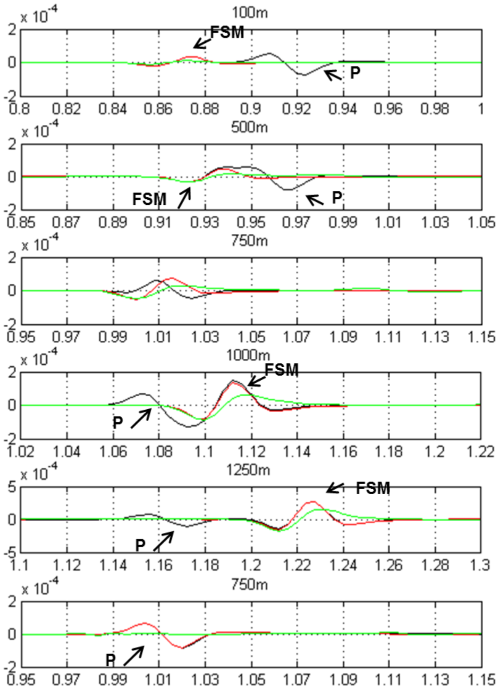

Figure 5: Result for the test which has the input data generated with absorption in the model. The top five traces show a trace comparison between the data and two predictions at different offsets. Black, red and green lines represent actual data (consisting of primary and free-surface multiple), ISS FSME prediction and SRME prediction, respectively. At offset 750m, the bottom trace shows the comparison between the actual primary (black) in the data and result after ISS (red) and SRME+adaptive (green) algorithm. The SRME + Adaptive damages the primary that interferes with the free surface multiple. The ISS free-surface algorithm effectively removes the free surface multiple without damaging the primary

DISCUSSION

Pre-requisites of the ISS FSME algorithm

As demonstrated, the ISS FSME has the distinct property that it can predict free-surface multiples with accurate time and accurate amplitude, and without any subsurface information.

For the ISS FSME algorithm to deliver its full capability, there are pre-requisites: source signature estimation, removal of reference wave, and source and receiver-side deghosting. Green's Theorem methods have been developed to achieve those pre-requisites. For example, Weglein and Secrest (1990) predicts the reference wave and the source wavelet; Weglein et al. (2002); Zhang (2007), Mayhan (2013), Wu and Weglein (2017), Zhang and Weglein (2016) and Shen and Weglein (2017) developed Green's theorem methods to surgically remove the reference wave (without damaging the reflection data) and to deghost and can accommodate a depth-variable cable. The impact of these pre-requisites on the effectiveness of ISS FSME are examined in, e.g., Zhang (2007); Yang and Weglein (2016).

CONCLUSION

In this abstract, we test and analyze the difference between the ISS FSME algorithm and the SRME plus energy-minimization adaptive subtraction. The test results show that to remove isolated free-surface multiples, SRME (with approximate time and amplitude prediction) plus energy minimization adaptive subtraction can be an appropriate choice. To remove interfering multiples without damaging primaries, the ISS method (with accurate time and amplitude prediction) is the appropriate choice when multiples interfere with other events. We provide the ISS method as an option in the multiple removal toolbox when this type of capability is called-upon.

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