M-OSRP
Misson - Oriented Seismic Research Program
Solve The Right Problem

Initial analysis and comparison of the amplitude properties/information of asymptotic and wave equation migration for one-way propagating waves: implications for RTM

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Austin, Texas

Outline

- Background
- Wave equation and asymptotic migration
- Test and comparison with the simplest case
- Conclusion
- Acknowledgement

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Background

- Methods that use the wave equation to perform seismic migration have two ingredients:
  1) wave propagation
  2) imaging principle or concept (imaging condition)

Imaging conditions

- There are three key imaging conditions :
  1) time and space coincidence of up-gong and down-going wave
  2) the exploding-reflector model
  3) predicting an experiment at a coincident-source-and-receiver subsurface point (image point), and asking for time equals zero

Imaging conditions

- For a normal-incident spike plane wave on a horizontal reflector, these three imaging concepts are totally equivalent.
- However, for a non-zero-offset surface seismic-data experiment they are no longer equivalent (for either a one-dimensional or a multi-dimensional subsurface)
- For determining quantitative information on the physical meaning of the image, the clear choice is predicting a source and receiver experiment at depth.

Wave equation migration

- Wave-equation migration (WEM) is defined as using the third imaging condition

- In anything beyond 1D normal-incidence or 1D zero-offset data, the other two imaging condition are asymptotic ray ("Kirchhoff") algorithms with a trajectory of image candidates by constructive summation.

Wave equation migration

- The benefits of imaging condition (3) comparing with other two
  - Definitiveness of a subsurface point corresponding to (or not corresponding to) structure from a predicted source and receiver experiment at that point;
  - Quantitative angle-dependent reflection coefficient information at the image point;
  - Widespread wave propagation and wave illumination, compared to limited propagation and illumination of asymptotic ray theory algorithms

Wave equation migration

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  - Asymptotic migration
  - Angle-dependent information from asymptotic migration
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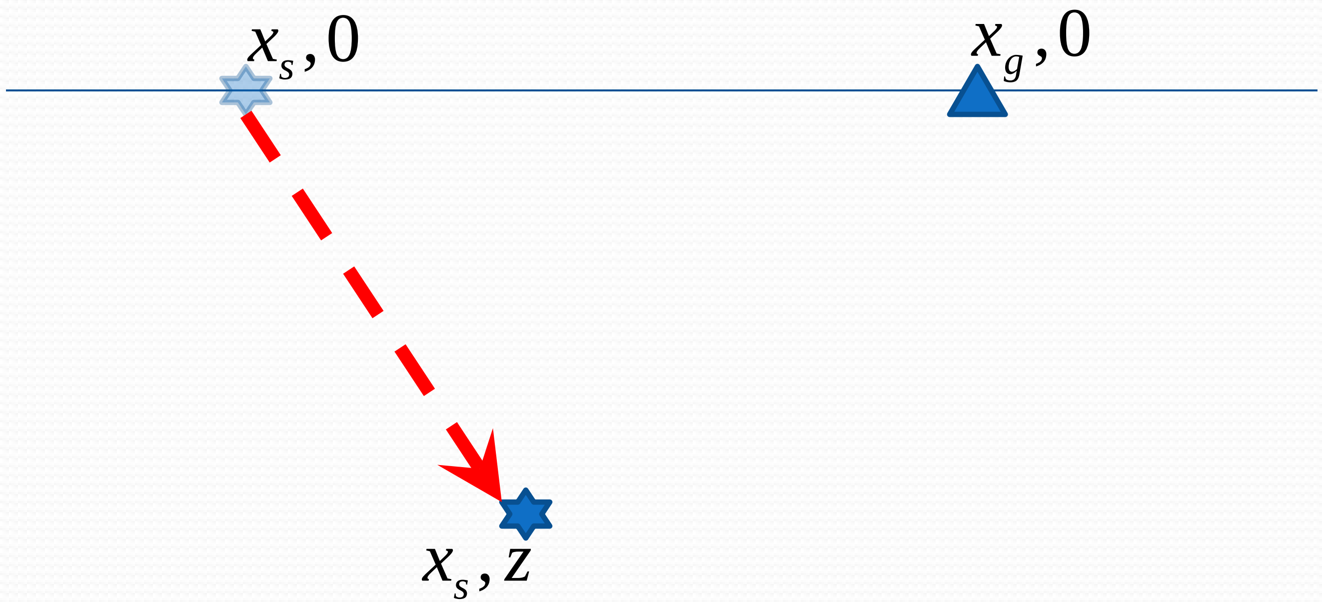
Wave-equation migration – step 1

$$D(x_g, 0 \mid x_s, 0; t)$$

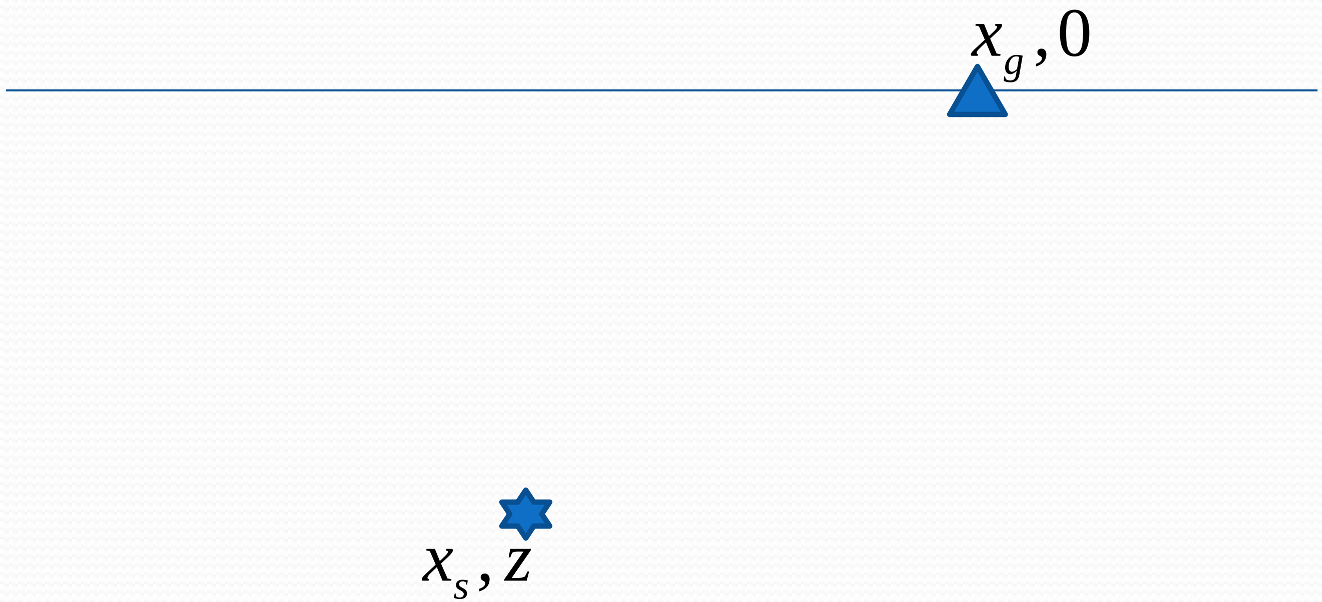
Wave-equation migration – step 1



$$D(x_g, 0 \mid x_s, 0; t)$$

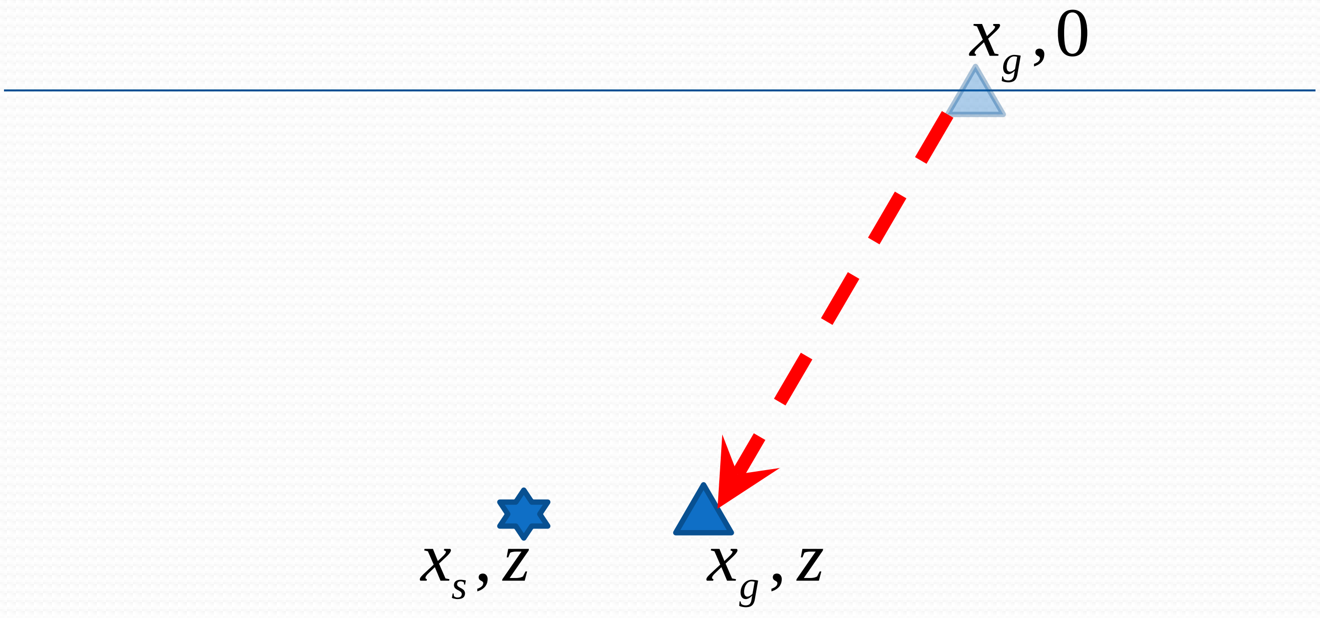
$$P(x_g, 0 \mid x_s, z; t)$$

Wave-equation migration – step 1



$$P(x_g, 0 \mid x_s, z; t)$$

Wave-equation migration – step 2



$$P(x_g, 0 \mid x_s, z; t)$$

$$P(x_g, z \mid x_s, z; t)$$

Wave-equation migration – step 2

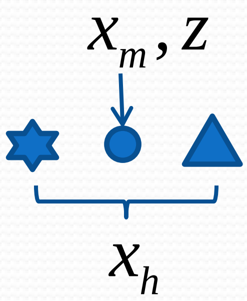
$$P(x_g, z \mid x_s, z; t)$$

x_s, z $\quad$ x_g, z

Wave-equation migration – step 3

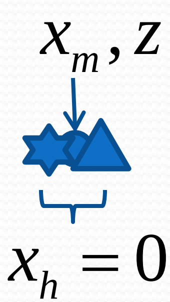
$$P(x_g, z \mid x_s, z; t)$$

$$= P(x_m, z, x_h; t)$$

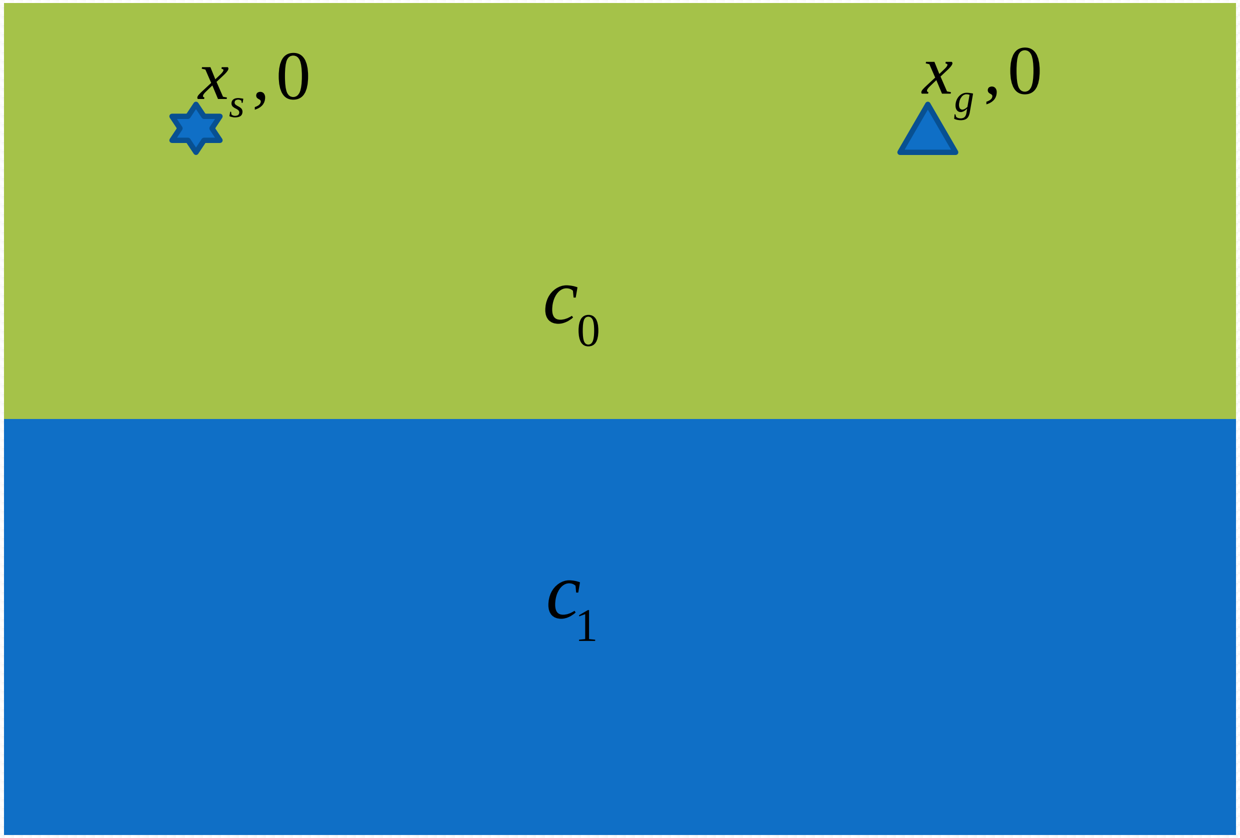


Wave-equation migration – step 3

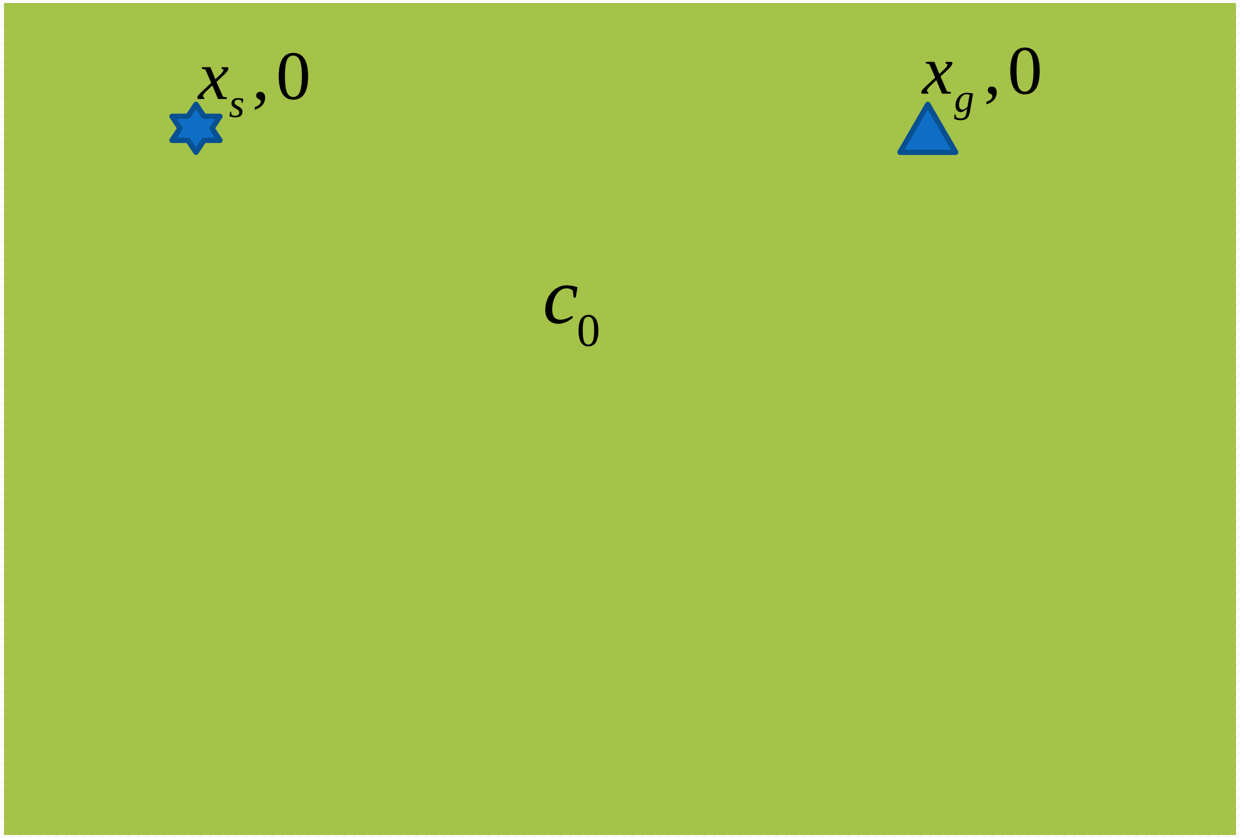
$$P(x_m, z, x_h = 0; t = 0)$$
$$= I(x_m, z)$$



The simplest WEM



The reference medium



The reference medium

- Acoustic wave equation in reference medium

$$\left(\nabla^2 + \frac{\omega^2}{c_0{}^2}\right) P(x, z, | x_s, z_s; \omega) = 0$$

- In reference medium and below the source and the receiver, the receiver wave field propagation is one-way (up-going only)

One-way wave-equation migration

- Given the one-way wave propagation and homogenous reference medium, the 3 steps of wave-equation migration can be implemented as
- Step 1: downward continuation of source

$$P(k_{gx},0|k_{sx},z;\omega) = D(k_{gx}|k_{sx};\omega)e^{-ik_{sz}z}$$

- Step 2: downward continuation of receiver (from reciprocity)

$$P(k_{gx},\mathrm{z}|k_{sx},\mathrm{z};\omega) = P(k_{gx},0|k_{sx},z;\omega)e^{ik_{gz}z}$$
$$= D(k_{gx}|k_{sx};\omega)e^{i(k_{gz}-k_{sz})z}$$

One-way wave-equation migration

- Step 3: apply image condition (source and receiver coincidence and t=0)

$$M(x,z) \equiv P(x_g = x, z_g = z | x_s = x, z_s = z; t = 0)$$

$$= \frac{1}{(2\pi)^3} \int d\omega \int dk_{sx} \int dk_{gx} D(k_{gx} | k_{sx}; \omega) e^{i(k_{gz} - k_{sz})z} e^{i(k_{gz} - k_{sz})x}$$

One-way wave-equation migration

- Change integral variables from k_{sx}, k_{gx}, ω to k_x, k_h, k_z

$$k_x = k_{gx} - k_{sx}; \quad k_h = k_{gx} + k_{sx}; \quad k_z = k_{gz} - k_{sz}$$

$$M(x,z) = \frac{c^2}{2(2\pi)^3} \int dk_z \int dk_x \int dk_h D(k_{gx} \mid k_{sx}; \omega) \frac{k_{sx} k_{gx}}{\omega k_z} e^{ik_z z} e^{ik_x x}$$

One-way wave-equation migration

- Remove 2 inverse Fourier transforms

$$M(x,z) = \frac{c^2}{2(2\pi)^3} \int dk_z \int dk_x \int dk_h D(k_{gx} \mid k_{sx}; \omega) \frac{k_{sx} k_{gx}}{\omega k_z} e^{ik_z z} e^{ik_x x}$$

$$M(k_x, k_z) = \frac{c^2}{4\pi} \int dk_h D(k_{gx} \mid k_{sx}; \omega) \frac{k_{sx} k_{gx}}{\omega k_z}$$

One-way wave-equation migration

- Remove 2 inverse Fourier transforms

$$M(x,z)=\frac{c^2}{2(2\pi)^3}\int dk_z\int dk_x\int dk_h D(k_{gx}\,|\,k_{sx};\omega)\frac{k_{sx}k_{gx}}{\omega k_z}e^{ik_z z}e^{ik_x x}$$

$$M(k_x,k_z)=\frac{c^2}{4\pi}\int dk_h D(k_{gx}\,|\,k_{sx};\omega)\frac{k_{sx}k_{gx}}{\omega k_z}$$

- Migration image with angle-dependent amplitude information (uncollapsed migration image)

$$M(k_x,k_z,k_h)=\frac{c^2}{4\pi}\frac{k_{sx}k_{gx}}{\omega k_z}D(k_{gx}\,|\,k_{sx};\omega)$$

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The simplest asymptotic migration - one-way asymptotic migration

- Begin with the one-way wave equation migration (also known as Stolt migration)

$$M(x,z)=\frac{c^2}{2(2\pi)^3}\int dk_z\int dk_x\int dk_h D(k_{gx}\,|\,k_{sx};\omega)\frac{k_{sx}k_{gx}}{\omega k_z}e^{ik_z z}e^{ik_x x}$$

$$=\frac{1}{(2\pi)^3}\int dx_g\int dx_s\int dt D(x_g\,|\,x_s;t)\int d\omega e^{i\omega t}$$

$$\times\int dk_{sx}\int dk_{gx}e^{-i(k_{sz}z+k_{sx}(\mathrm{x}-x_s))}e^{i(k_{gz}z+k_{gx}(\mathrm{x}-x_g))}$$

The simplest asymptotic migration - one-way asymptotic migration

- Stationary phase approximations

$$\int dk_{sx} e^{-i(k_{sz}z + k_{sx}(x - x_s))} \simeq e^{i\omega r_s/c} \sqrt{\frac{2\pi i\omega z^2}{c r_s{}^3}}$$

$$\int dk_{gx} e^{i(k_{gz}z + k_{gx}(x - x_g))} \simeq e^{i\omega r_g/c} \sqrt{\frac{2\pi i\omega z^2}{c r_g{}^3}}$$

where

$$r_s = \sqrt{z^2 + (x - x_s)^2}$$

$$r_g = \sqrt{z^2 + (x - x_g)^2}$$

The simplest asymptotic migration - one-way asymptotic migration

- We get the one-way asymptotic migration

$$M^{A}(x,z) = \frac{z^{2}}{(2\pi)^{2}c}\int dx_{s}\int dx_{g}\,\frac{\frac{\partial}{\partial t}D(x_{g}\,|\,x_{s};t=r/c)}{(r_{g}r_{s})^{3/2}}$$

$$= \frac{z^{2}}{(2\pi)^{2}c}\int d\tilde{x}_{h}\int d\tilde{x}_{m}\,\frac{\frac{\partial}{\partial t}D(\tilde{x}_{m},\tilde{x}_{h};t=r/c)}{(r_{g}r_{s})^{3/2}}; \quad r = r_{s}+r_{g}$$

One-way asymptotic migration is a Kirchhoff-like ray-based method

- This asymptotic migration formula is a weighted summation of the data along a trajectory of travel-times corresponding to ray-paths from the source to image point and then to receiver

- A general ray theory based Kirchhoff migration formula is

$$I(x,z)=\int d\tilde{x}_h \int d\tilde{x}_m W(x,z,\tilde{x}_m,\tilde{x}_h) D(\tilde{x}_m,\tilde{x}_h,t=r/c)$$

where the W is a weighting function

One-way asymptotic migration is a Kirchhoff-like ray-based method

- The asymptotic migration is derived as an asymptotic approximation of WEM for one-way waves
- It is a prototype of Kirchhoff migration

$$M^A(x,z) = \frac{z^2}{(2\pi)^2 c} \int d\tilde{x}_h \int d\tilde{x}_m \frac{\frac{\partial}{\partial t} D(\tilde{x}_m, \tilde{x}_h; t = r/c)}{(r_g r_s)^{3/2}}$$

$$I(x,z) = \int d\tilde{x}_h \int d\tilde{x}_m W(x,z,\tilde{x}_m,\tilde{x}_h) D(\tilde{x}_m,\tilde{x}_h,t = r/c)$$

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Angle-dependent information from asymptotic one-way migration

- $M^A(x,z)$ does not come from a coincident (zero offset) source-receiver experiment at depth
- However, we want to obtain angle-dependent amplitude information for asymptotic migration

Assistance for asymptotic one-way migration

- Interpret $M^{A}(x,z)$ as though it was the output of an imagined or fictitious zero offset experiment at t=0

$$M^{A}(x,z) \triangleq M^{A}(x_m, z, x_h = 0)$$

- Assume by causality that for $x_h \neq 0$ at t=0 the measurement would be zero

$$M^{A}(x_m, z_m, x_h) \triangleq 0 \quad \text{for} \quad x_h \neq 0.$$

- We can compare M with M^{A} in terms of angle-dependent amplitude information

Alternative way for angle-dependent amplitude information

- Alternatively, we may use another way to obtain angle-dependent amplitude information for asymptotic migration
- That is constant offset partial asymptotic migration in an attempt to deduce angle-dependent information at the output point

Partial asymptotic migration for angle-dependent information

- Uncollapsed Wave equation migration

$$M(k_x, k_z) = \frac{c^2}{4\pi^2} \int dk_h D(k_{gx} \mid k_{sx}; \omega) \frac{k_{sx} k_{gx}}{\omega k_z}$$

$$M(k_x, k_z, k_h) = \frac{c^2}{4\pi^2} \frac{k_{sx} k_{gx}}{\omega k_z} D(k_{gx} \mid k_{sx}; \omega)$$

Partial asymptotic migration for angle-dependent information

- Uncollapsed Wave equation migration

$$M(k_x,k_z)=\frac{c^2}{4\pi^2}\int dk_h D(k_{gx}\,|\,k_{sx};\omega)\frac{k_{sx}k_{gx}}{\omega k_z}$$

$$M(k_x,k_z,k_h)=\frac{c^2}{4\pi^2}\frac{k_{sx}k_{gx}}{\omega k_z}D(k_{gx}\,|\,k_{sx};\omega)$$

- Partial asymptotic migration

$$M^A(x,z)=\frac{z^2}{(2\pi)^2 c}\int d\tilde{x}_h\int d\tilde{x}_m\frac{\frac{\partial}{\partial t}D(\tilde{x}_m,\tilde{x}_h;t=r/c)}{(r_g r_s)^{3/2}}$$

$$M^A(x,z,\tilde{x}_h)=\frac{z^2}{(2\pi)^2 c}\int d\tilde{x}_m\frac{\frac{\partial}{\partial t}D(x_g\,\big|\,x_s;t=r/c)}{(r_g r_s)^{3/2}}$$

Partial asymptotic migration for angle-dependent information

- we could treat the partial asymptotic migration

$$M^A(x, z, \tilde{x}_h) = \frac{z^2}{(2\pi)^2 c} \int d\tilde{x}_m \frac{\frac{\partial}{\partial t} D(x_g \mid x_s; t = r/c)}{(r_g r_s)^{3/2}}$$

as though it corresponded to x_h in $M^A(x_m, z_m, x_h)$ (although it doesn't).

Partial asymptotic migration for angle-dependent information

- However, we will show this idea does not work even for the simplest possible imaging example

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A test with the simplest scenario

- We will demonstrate the differences between the wave equation migration and the asymptotic migration in terms of the amplitude in a simplest possible model with offset data.

	Wave-equation migration	Asymptotic migration
Migration Method	One-way Stolt migration	Asymptotic migration of one-way Stolt migration
Data	analytic data (Reflectivity method method in wavenumber-frequency domain)	analytic data (Cagniard-de Hoop method in space-time domain)
Migration procedure	Step 1: downward continuation of source	Step 1: start with one-way Stolt migration formula
	Step 2: downward continuation of receiver	Step 2: apply stationary phase approximations
	Step 3: apply image condition	
Imaging result	$M(x,z)$	$M^A(x,z)$
Imaging result with subsurface offset	$M(k_m , k_h , k_z)$ or $M(x , x_h , z)$	N/A (requires introduction of an imaginary experiment corresponding to the asymptotic imaging result)

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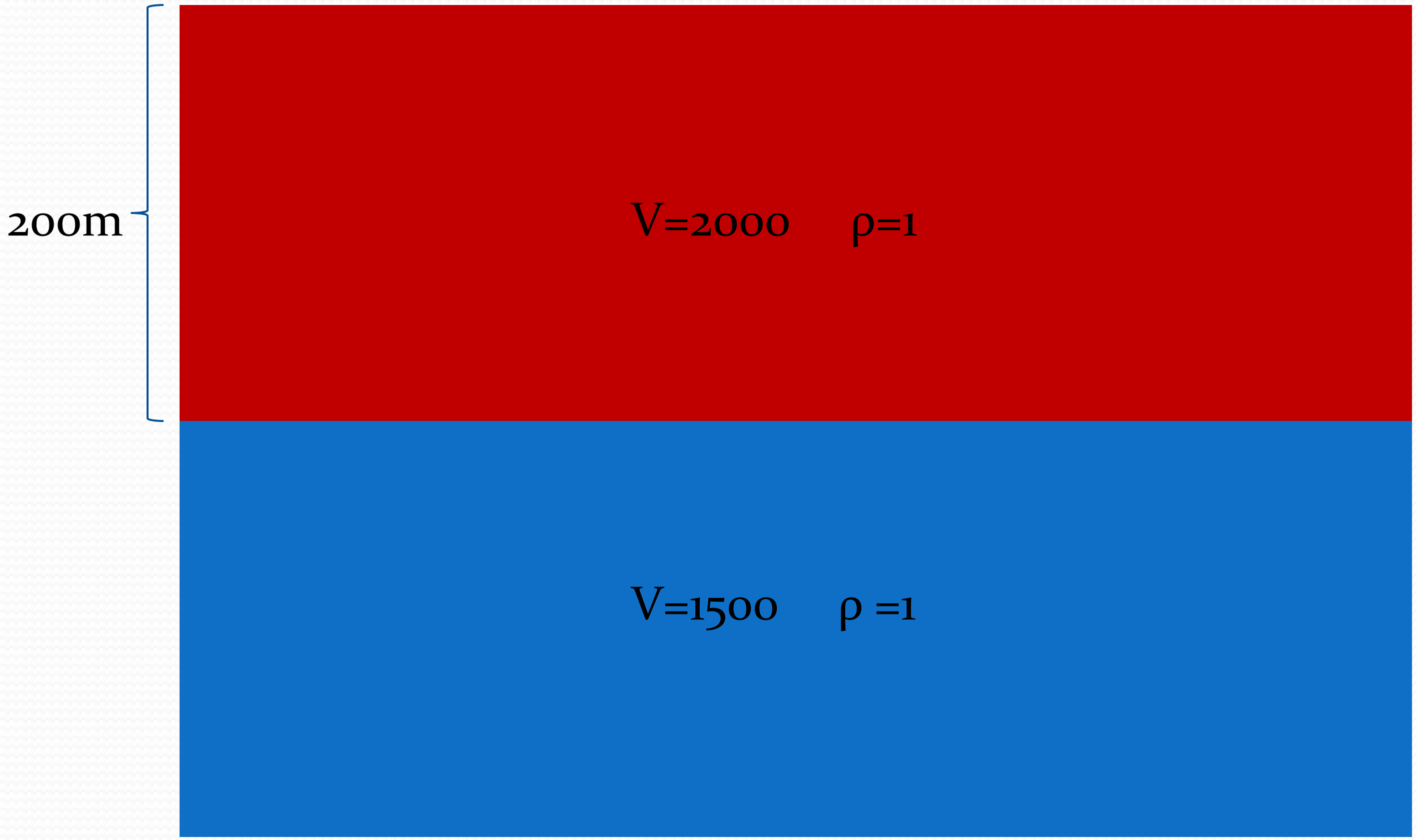
The model



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The data for the test

- We use analytic data for both wave-equation migration and asymptotic migration .
- Using analytic data will avoid the effect of any numerical inaccuracy in data generating procedure.

Data for wave-equation migration

- One-way wave-equation migration

$$M(k_x, k_z) = \frac{c^2}{4\pi} \int dk_h D(k_{gx} \mid k_{sx}; \omega) \frac{k_{sx} k_{gx}}{\omega k_z}$$

- Analytic data generated by reflectivity method
- A 2D source and a 1D earth
- e.g. in Ewing, Jardetzky and Press (1957)

$$D(k_S, k_g, \omega) = \delta(k_s - k_g) \frac{r(k_s, k_{zs}) e^{ik_s x_g} e^{2iq_s z_r}}{4\pi i k_{zs}}$$

$$r(k_s, k_{sz}) = \frac{\rho_2 k_{sz1} - \rho_1 k_{sz2}}{\rho_2 k_{sz1} + \rho_1 k_{sz2}}$$

Data for asymptotic migration

- One-way asymptotic migration

$$M^A(x,z)=\frac{z^2}{(2\pi)^2 c}\int d\tilde{x}_h \int d\tilde{x}_m \frac{\frac{\partial}{\partial t} D(\tilde{x}_m,\tilde{x}_h;t=r/c)}{(r_g r_s)^{3/2}}; \quad r=r_s+r_g$$

- Analytic data generated by Cagniard-de Hoop method
- A 2D source and a 1D earth
- Utilized in Jingfeng and Weglein (2005)

$$D(x_m,x_h,t)=\frac{1}{2\pi}\mathrm{Re}(\widehat{pp})\frac{H(t-R/c_0)}{\sqrt{t-R^2/c_0{}^2}}$$

$$R=\sqrt{(x_s-x_g)^2+(z_s+z_g-2z_r)^2}$$

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The results

	Asymptotic migration	Wave-equation migration
(x_h,z) domain	$M^A(x_m, x_h, z)$	
(k_h,z) domain		
(k_h, k_z) domain		$M(k_m, k_h, k_z)$

The results

	Asymptotic migration	Wave-equation migration
(x_h,z) domain	$M^A(x_m , x_h , z)$	
(k_h,z) domain		
(k_h, k_z) domain		$M(k_m , k_h , k_z)$

The results

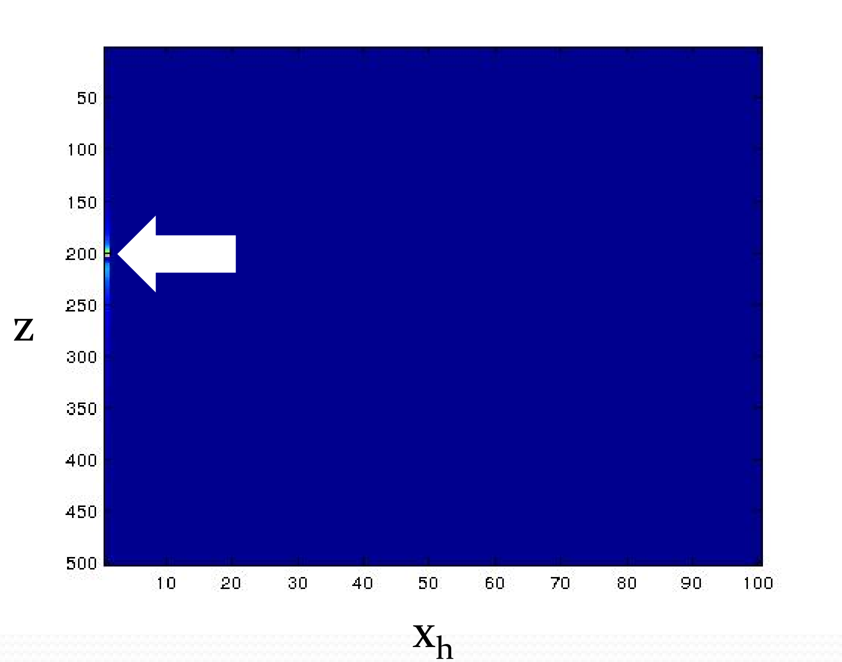
	Asymptotic migration	Wave-equation migration
(x_h,z) domain	$M^A(x_h , z)$	$M(x_h , z)$
	↓ F.T.	↑ I.F.T.
(k_h,z) domain	$M^A(k_h , z)$	$M(k_h , z)$
	↓ F.T.	↑ I.F.T.
(k_h, k_z) domain	$M^A(k_h , k_z)$	$M(k_h , k_z)$

The results

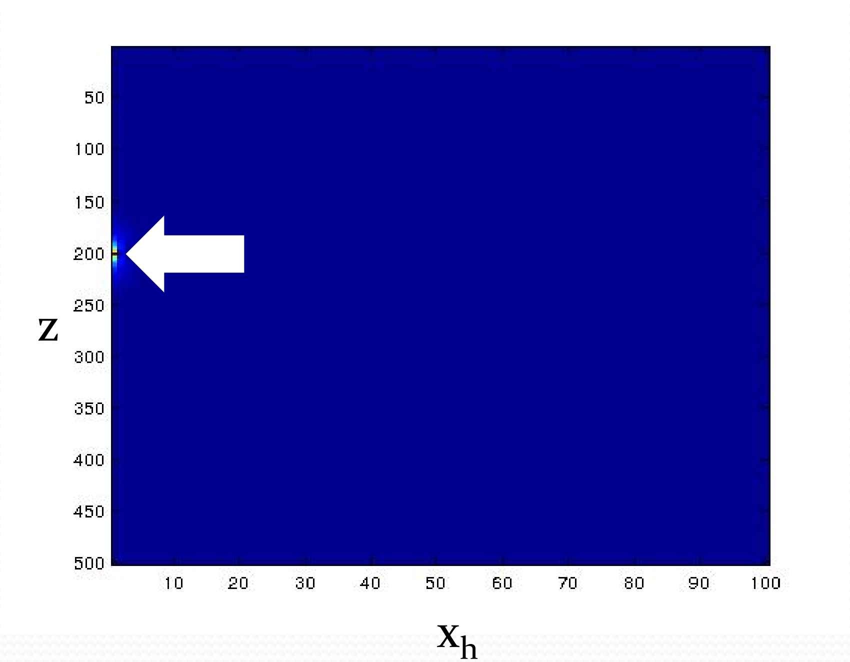
	Asymptotic migration	Wave-equation migration
(x_h,z) domain	$M^A(x_h, z)$	$M(x_h, z)$
	↓ F.T.	↑ I.F.T.
(k_h,z) domain	$M^A(k_h, z)$	$M(k_h, z)$
	↓ F.T.	↑ I.F.T.
(k_h, k_z) domain	$M^A(k_h, k_z)$	$M(k_h, k_z)$

Image in (x_h,z)

Asymptotic



Wave-equation

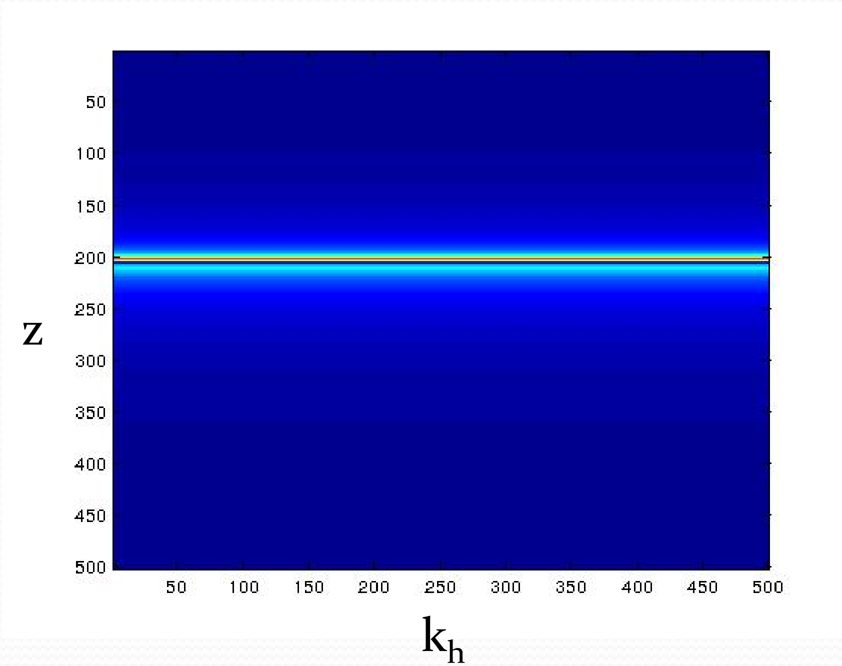


The results

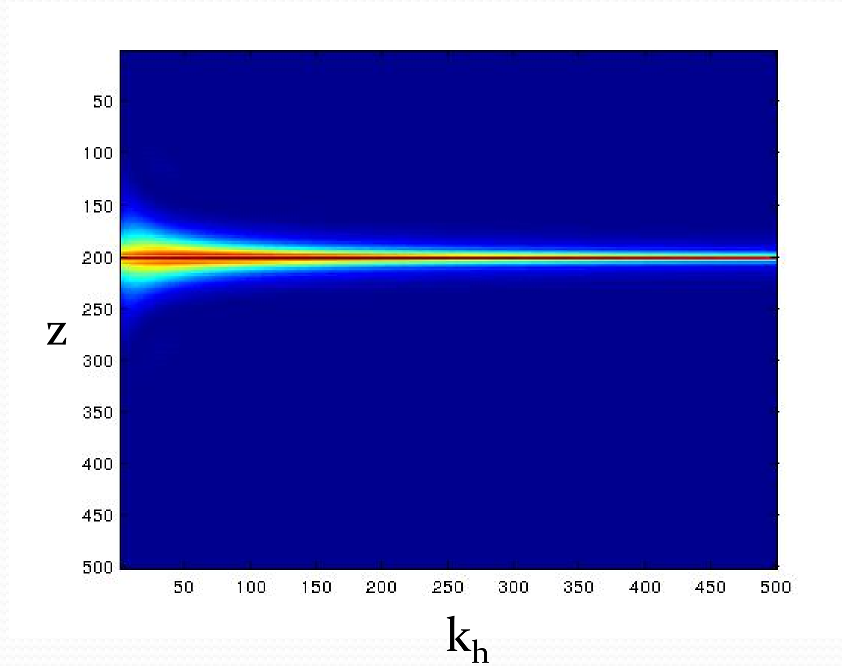
	Asymptotic migration	Wave-equation migration
(x_h,z) domain	$M(x_h , z)$	$M^A(x_h , z)$
	↓ F.T.	↑ I.F.T.
(k_h,z) domain	$M(k_h , z)$	$M^A(k_h , z)$
	↓ F.T.	↑ I.F.T.
(k_h, k_z) domain	$M(k_h , k_z)$	$M^A(k_h , k_z)$

Image in (k_h,z)

Asymptotic



Wave-equation



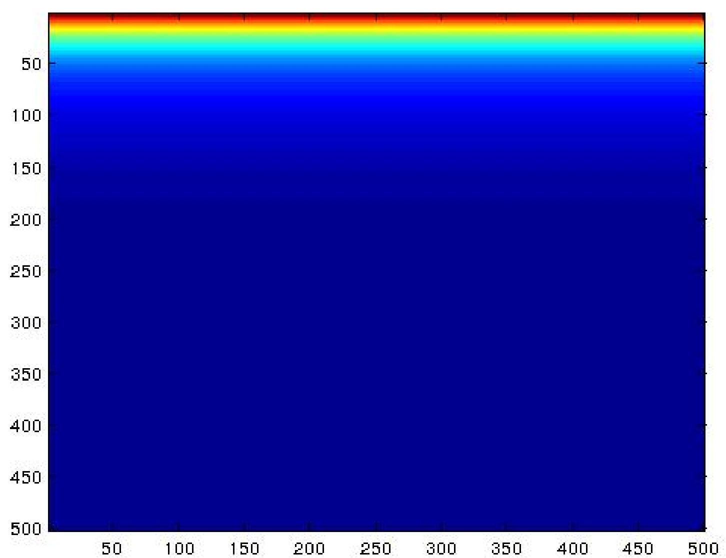
The results

	Asymptotic migration	Wave-equation migration
(x_h,z) domain	$M(x_h , z)$	$M^A(x_h , z)$
	↓ F.T.	↑ I.F.T.
(k_h,z) domain	$M(k_h , z)$	$M^A(k_h , z)$
	↓ F.T.	↑ I.F.T.
(k_h, k_z) domain	$M(k_h , k_z)$	$M^A(k_h , k_z)$

Image in (k_z, k_h)

Asymptotic

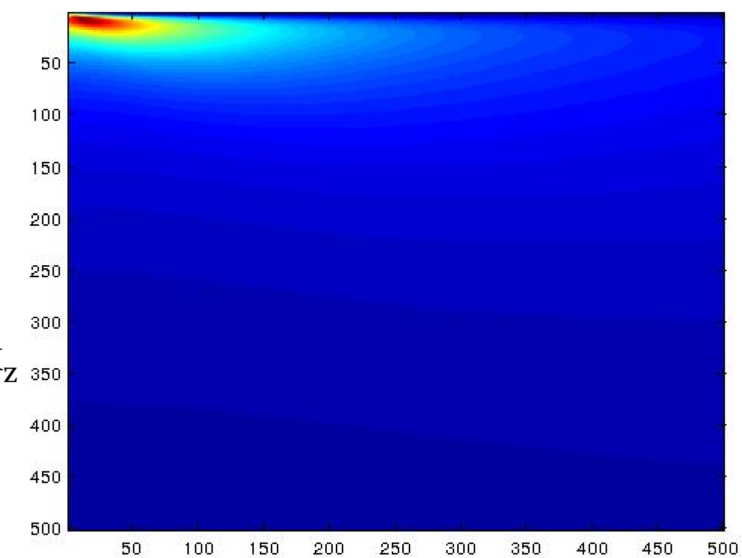
k_z



k_h

Wave-equation

k_z



k_h

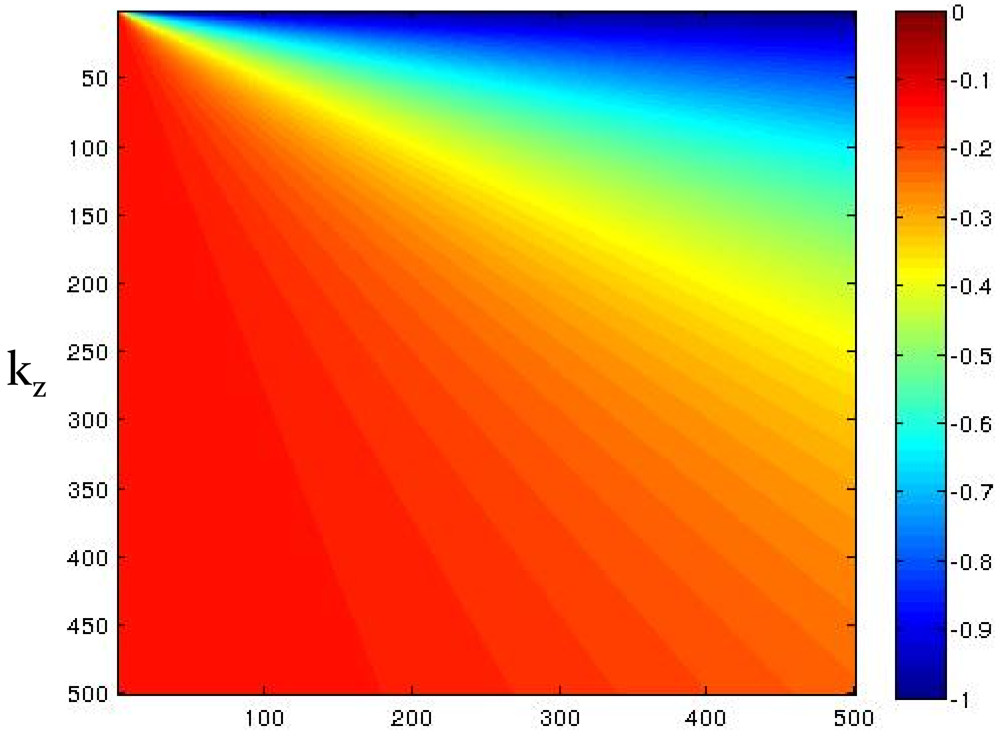
The reflection coefficient from image

- Invert the image amplitude for angle-dependent reflection coefficient

$$r(k_x, k_z) = -\frac{\omega}{c^2}\frac{4\pi i}{e^{2ik_{sz}z_r}}M(k_h, k_z)$$

$$\left|r(k_x, k_z)\right| = \left|\frac{\omega 4\pi}{c^2}\right|\left|M(k_h, k_z)\right|$$

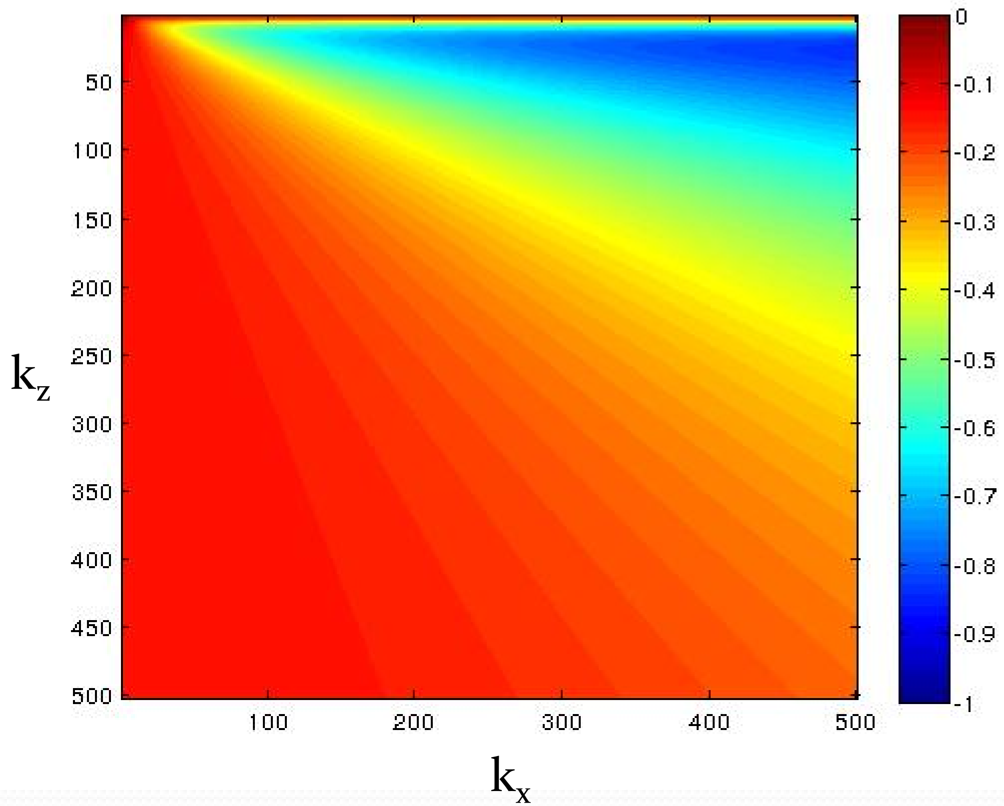
The analytic reflection coefficient



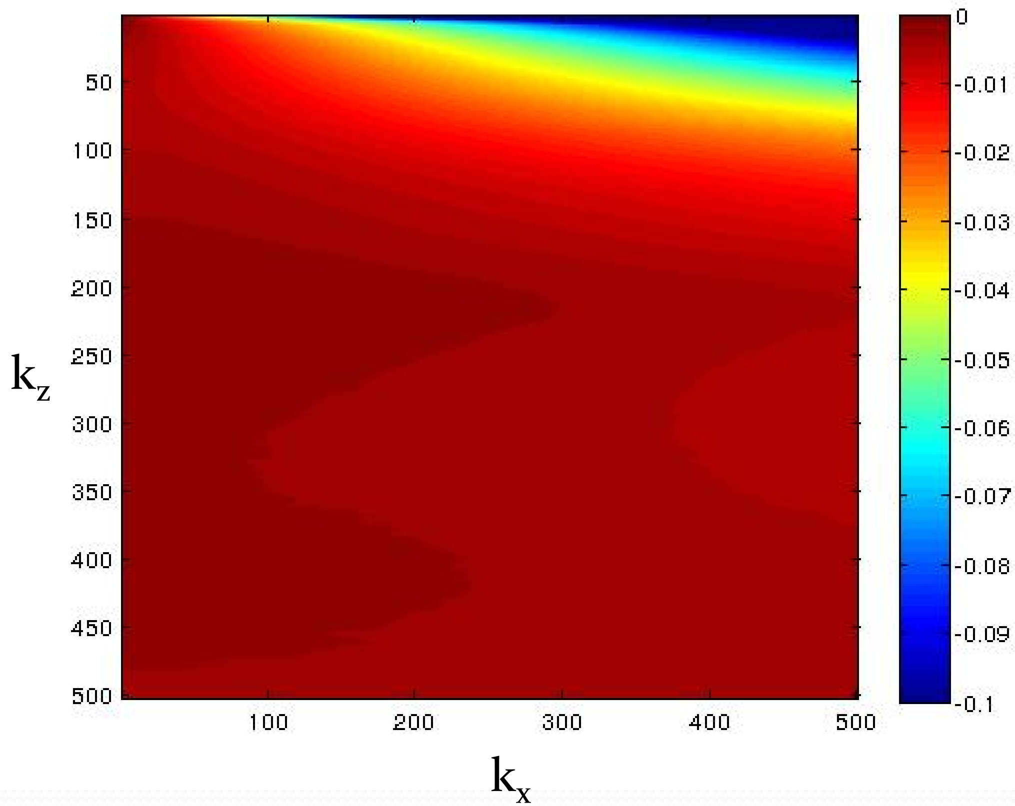
k_x

$$r(k_x, k_z) = \frac{\rho_2 k_{z1} - \rho_1 k_{z2}}{\rho_2 k_{z1} + \rho_1 k_{z2}}$$

The reflection coefficient from WEM



The reflection coefficient from Asymptotic migration



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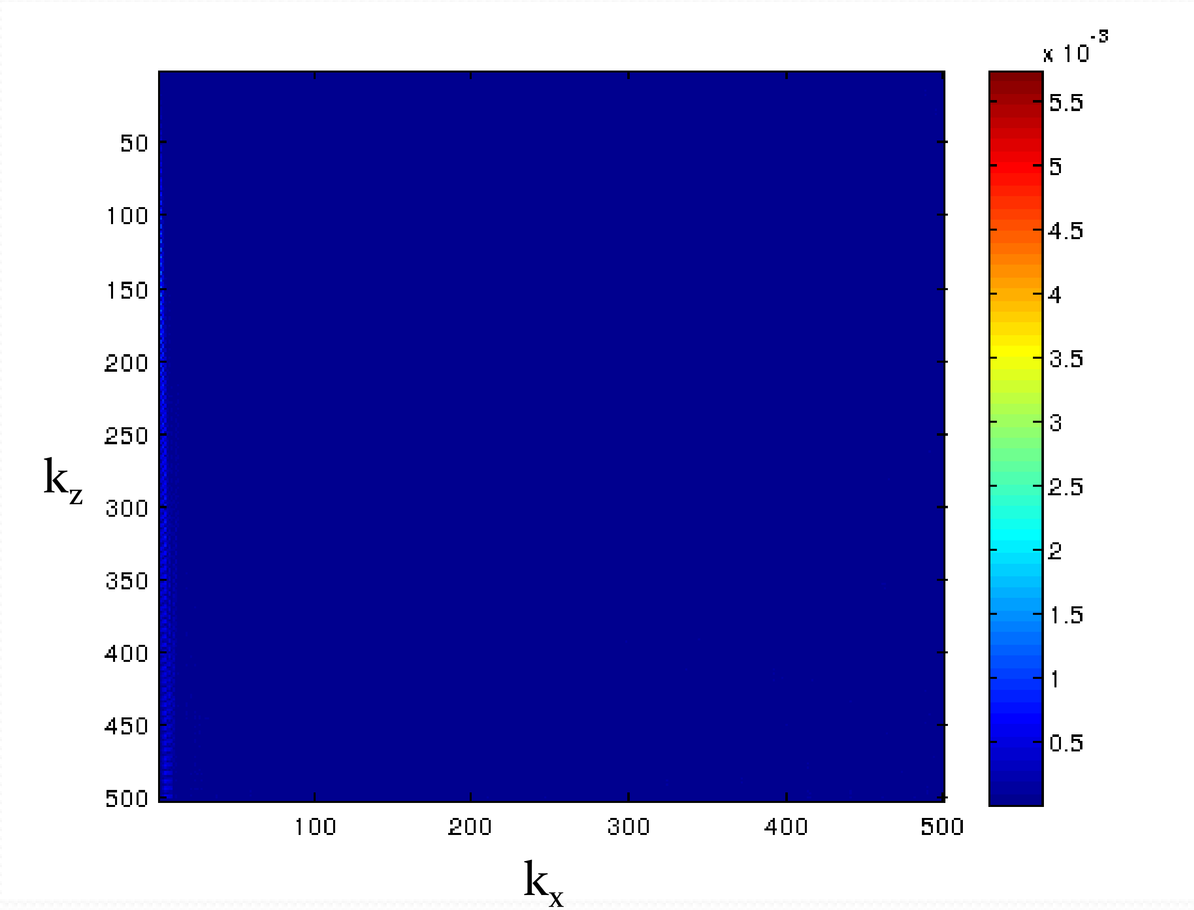
Partial asymptotic migration for angle-dependent information

- we could treat the partial asymptotic migration

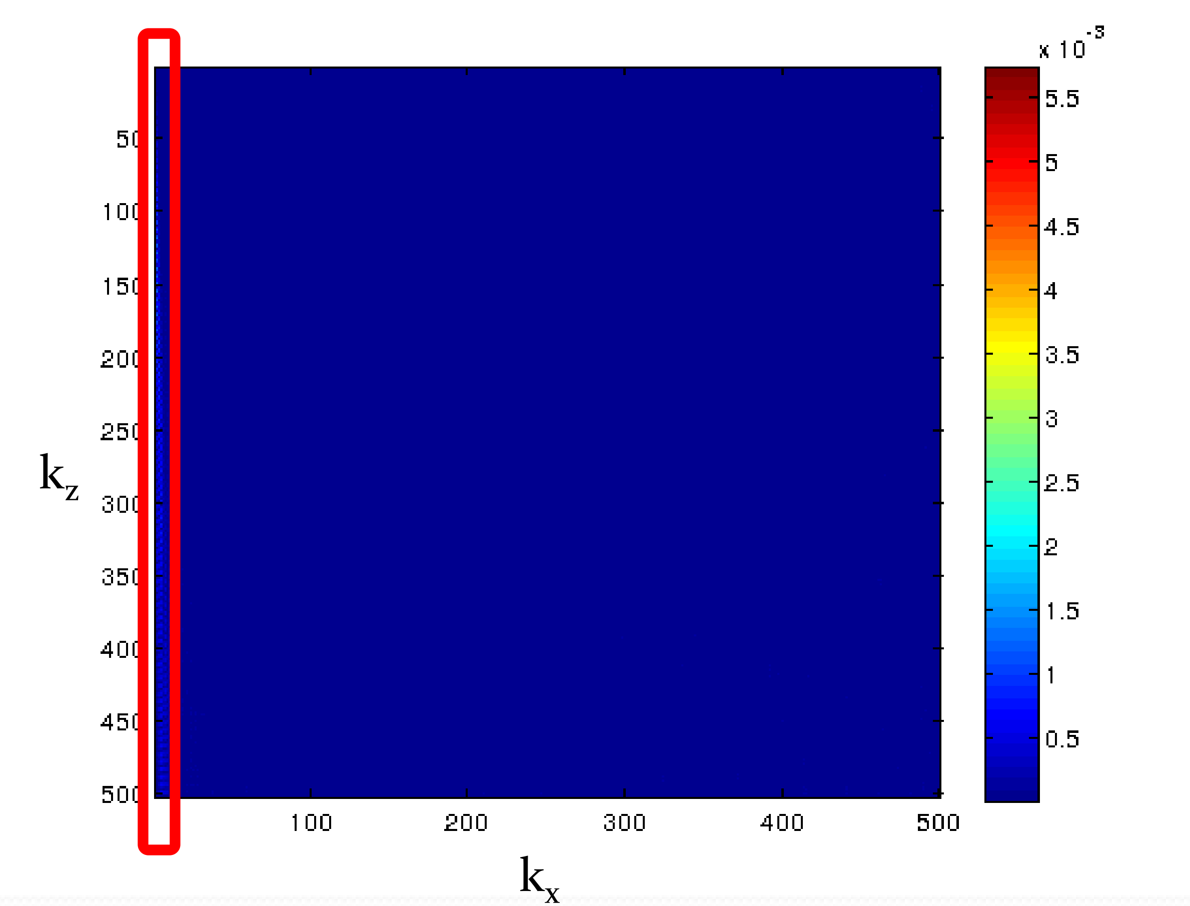
$$M^{A}(x, z, \tilde{x}_h) = \frac{z^2}{(2\pi)^2 c} \int d\tilde{x}_m \frac{\frac{\partial}{\partial t} D(x_g \middle| x_s; t = r / c)}{(r_g r_s)^{3/2}}$$

as though it corresponded to x_h in $M^{A}(x_m, z_m, x_h)$ (although it doesn't).

Partial asymptotic migration for angle-dependent information



Partial asymptotic migration for angle-dependent information



Outline

- Background
- Wave equation and asymptotic migration
- Test and comparison with the simplest case
- Conclusion
- Acknowledgement

Conclusions

- We compare the amplitudes of the wave equation migration and the asymptotic migration for the simplest possible overburden, using both perfect data in the domain each requires, and seek to retrieve the angle-dependent reflection coefficient from both migration methods.
- The amplitude of wave-equation migration is closely related to the angle-dependent reflection coefficient of the reflector, thus angle-dependent reflection coefficient can be retrieved from wave-equation migration imaging.
- The asymptotic approximation does not correspond to a coincident source and receiver experiment at depth. We cannot retrieve the angle-dependent reflection coefficient after the asymptotic approximation is applied to the the wave equation migration method.

Conclusions

- Attempts to interpret the asymptotic migration result as due to an imaginary source and receiver experiment at the image point, does not provide the angle-dependent reflection coefficient.

- The common practice (in asymptotic migration include current RTM) of calling upon constant offset partial migrations in an attempt to deduce angle-dependent information at the output point is shown in this report (even in this simplest possible imaging problem) to not correspond to the angle-dependent reflection coefficient at the output point.

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Thank you