M-OSRP
Misson - Oriented Seismic Research Program
Solve The Right Problem

Short note: An alternative adaptive subtraction criteria (to energy minimization) for free surface multiple removal

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May 27-30, 2014
Austin, TX

An idea: The idea starts by considering the following perturbation forms in 3D.

$$
\begin{aligned}
&\text{1 parameter acoustic} \quad V_p \\
=&\omega^2 \left(\frac{1}{K_0} a_1(\vec{k}-\vec{p}) \right) = V_p(\vec{k},\vec{p},\omega) \\
&\text{2 parameter acoustic} \quad V_{pp} \\
=&\omega^2 \left(\frac{1}{K_0} a_1(\vec{k}-\vec{p}) + \frac{1}{\omega^2} \frac{\vec{k}\cdot\vec{p}}{\rho_0} a_2(\vec{k}-\vec{p}) \right)
\end{aligned}
$$

elastic V_{pp}

$$=\omega^2 \left(\frac{1}{K_0} a_1(\vec{k} - \vec{p}) + \frac{1}{\omega^2} \frac{\vec{k} \cdot \vec{p}}{\rho_0} a_2(\vec{k} - \vec{p}) \right.$$

$$\left. - 2\frac{\beta_0^2}{\omega^4} |\vec{k} \times \vec{p}|^2 a_3(\vec{k} - \vec{p}) \right)$$

where $\vec{k}$ and $\vec{p}$ are arbitrary 3D vectors, K_0, ρ_0, and β_0 are the bulk modulus, density, and shear velocity of the reference medium, a_1 is the relative change in the bulk modulus, a_2 is the relative change in density, and a_3 is the relative change in shear modulus. On the measurement surface, $\vec{k} = \vec{k}_g$, $\vec{p} = \vec{k}_s$, and $|\vec{k}_g| = |\vec{k}_s| = \omega/c_0$. The input to the inverse scattering series free surface multiple algorithm is $V_1(\vec{k}_g, \vec{k}_s, \omega)$, which for the three forms listed above has an overall ω^2/c^2 factor.

$$D^{WOFS} = \underbrace{G_0^d V_1 G_0^d}_{\omega^2} + \underbrace{G_0^d V_1 G_0^{FS} V_1 G_0^d}_{\omega^4} + \underbrace{G_0^d V_1 G_0^{FS} V_1 G_0^{FS} V_1 G_0^d}_{\omega^6} + \ldots$$

where D^{WOFS} is deghosted data without free surface multiples and the first term on the right hand side is D^{WFS}, deghosted data with free surface multiples.

- The ISS free-surface-multiple algorithm has the following form (in 2D) (Carvalho, 1992):

$$D'(k_g, k_s, \omega) = \sum_{n=1}^{\infty} D'_n(k_g, k_s, \omega), \text{ where}$$

$$D'_n(k_g, k_s, \omega) = \frac{1}{i\pi\rho_0 A(\omega)} \int_{-\infty}^{\infty} dk\, q \exp\left(iq(\epsilon_g + \epsilon_s)\right) \times D'_1(k_g, k, \omega) D'_{n-1}(k, k_s, \omega).$$

- Summing terms D'_1 through D'_n removes free-surface multiples of orders 1 through $n-1$
— and alters higher order multiples.

The new adaptive criteria is

$$\lim_{\omega \to 0} \left[\frac{D^{WFS} - D^{WOFS}}{\omega^2} \right] = 0$$

or minimize $(D^{WFS} - D^{WOFS})/\omega^2$ with respect to a "wavelet" factor.

References I

Carvalho, Paulo Marcos. *Free-surface multiple reflection elimination method based on nonlinear inversion of seismic data*. PhD thesis, Universidade Federal da Bahia, 1992. In Portuguese.