Comparison and analysis of space and temporal frequency, and, spatial wavenumber and temporal frequency (e.g., P-$V_z$) domain approaches of Green's theorem de-ghosting techniques: Implications for 3D de-ghosting

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Key Points

• In 3D acquisition, the data in cross-line are typically characterized with sparse sampling and narrow aperture compared to in-line direction;

• Compared to spatial wavenumber and temporal frequency domain (e.g., P-V_\text{z} ) approach, Green’s theorem de-ghosting method achieved in space and temporal frequency domain shows the advantages of producing effective result and boosting low frequency energy.

• Numerical comparisons
  1) Spatial sampling interval
  2) Aperture
Outline

• Introduction and Motivation

• Theoretical Analysis

• Numerical Analysis
  o Spatial Sampling Interval
  o Aperture

• Conclusion
Motivation

• Ghosts:
  – (1) Cause notches in the frequency spectrum, especially for deep water acquisition, like OBC;
  – (2) Reduce the resolution, increase the uncertainty of inversion and interpretation.

• For our group, we wish to use isolated data for each processing step in order to get a more satisfactory result.
Introduction

Wegelein et al. (02), Zhang and Wegelein (05, 06); Zhang (07), Mayhan(12, 13):

Green’s theorem deghosting method

– Space and temporal frequency domain
– Spatial wavenumber and temporal frequency domain
Introduction

Weglein et al. (02), Zhang and Weglein (05, 06); Zhang (07), Mayhan (12, 13):

**Green’s theorem deghosting method**

- Space and temporal frequency domain
- Spatial wavenumber and temporal frequency domain

So question is:

Are they equivalent except calculated in different domains?
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• Introduction and Motivation

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  o Spatial Sample Interval
  o Aperture and Edge Effect

• Conclusion
Theoretical Analysis

- Green’s Theorem de-ghosting \((x - \omega)\)

\[
P'_R(x, z, x_s, z_s, \omega) = \int_{m.s.} (P(x', z', x_s, z_s, \omega)\nabla G_0^d(x, z, x', z', \omega) - G_0^d(x, z, x', z', \omega)\nabla P(x', z', x_s, z_s, \omega)) \cdot nds'
\]

- \(P'_R\) the receiver side de-ghosted data;
- \(P\) the pressure data;
- \(\nabla P\) the gradient of pressure data;
- \(G_0^d\) the causal Green’s function;
- \((x', z')\) point on measurement surface; \((x, z)\) prediction location;
- \((x_s, z_s)\) source location, \(\omega\) circular frequency.
Theoretical Analysis

• Green’s Theorem de-ghosting \((k_x - \omega)\)

  - Assume the acquisition geometry is horizontal

\[ P'_R(x, z, x_s, z_s, \omega) = \frac{1}{2} [P(k_x, z, x_s, z_s, \omega) - \frac{1}{i k_z} \frac{dP}{dz}(k_x, z, x_s, z_s, \omega)] \]

  - \(P'_R\) the receiver side de-ghosted data;
  - \(P\) the pressure data;
  - \(\frac{dP}{dz}\) the vertical derivative of pressure;
  - \(k_x\) horizontal wavenumber; \(k_z\) vertical wavenumber;
  - \((x, z)\) prediction location; \((x_s, z_s)\) source location,
  - \(\omega\) circular frequency.
If acquisition geometry is horizontal, Green’s theorem de-ghosting methods in these two different domains are theoretically equivalent.
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Even though geometry is horizontal, for cross-line, because of sparse spatial sampling and narrow aperture, spatial Fourier Transform will encounter difficulties to give a precise result.
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Numerical Analysis

-----Spatial Sampling Interval

• In order to prevent alias, the sampling interval should satisfy

\[ \Delta x \leq \frac{1}{2k_{\text{max}}} \quad \Delta t \leq \frac{1}{2f_{\text{max}}} \]

• Since

\[ k_{\text{max}} = \frac{f_{\text{max}}}{c} \]

• Then

\[ \Delta x \leq c / 2f_{\text{max}} \]

• If not, alias will appear in the data and contaminate the result.

So we need low-pass filtering before de-ghosting.
Air-water boundary

Vel=1500m/s  Depth=300m

Source: 7m
Receiver: 11m
Sampling interval: 3m
Aperture: 2400m

Velocity model
Keep the aperture of 2400m

Increase spatial sampling interval gradually: 3m – 12m – 30m – 60m – 100m

To reduce the space alias, apply low pass filter before calculation.
Aperture: 2400m
Spatial sampling interval: 3m

CdH upgoing wave

$k_x - \omega$ deghosted result

$x - \omega$ deghosted result
Spatial sampling interval: 3m

Red line: CdH upgoing wave
Blue line: $k_x - \omega$ deghosted
Spatial sampling interval: 3m

Red line: CdH upgoing wave
Green line: x - ω deghosted
Spatial sampling interval: 12m  (low cut filter: 60Hz)

Red line: CdH upgoing wave
Blue line: $k_x - \omega$ deghosted
Spatial sampling interval: 12m  (low cut filter: 60Hz)

Red line:    CdH upgoing wave
Green line:  x - ω deghosted
Spatial sampling interval: 30m (low cut filter: 25Hz)

Red line: CdH upgoing wave
Blue line: $k_x - \omega$ deghosted
Spatial sampling interval: 30m  (low cut filter: 25Hz)

Red line: CdH upgoing wave
Green line: $x - \omega$ deghosted
Spatial sampling interval: 60m  (low cut filter: 12Hz)

Red line: CdH upgoing wave
Blue line: $k_x - \omega$ deghosted
Spatial sampling interval: 60m  (low cut filter: 12Hz)

Red line: CdH upgoing wave
Green line: x - ω deghosted
Spatial sampling interval: **100m** (low cut filter: **7Hz**)

Red line: CdH upgoing wave
Blue line: $k_x - \omega$ deghosted
Spatial sampling interval: **100m** (low cut filter: 7Hz)

Red line: CdH upgoing wave
Green line: $x - \omega$ deghosted
<table>
<thead>
<tr>
<th>Spatial Sampling</th>
<th>$k_x - \omega$ domain result</th>
<th>$x - \omega$ domain result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dense</td>
<td>Ideal</td>
<td>Ideal</td>
</tr>
<tr>
<td>(e.g. 3m, 12m)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intermediate</td>
<td>Has residual</td>
<td>Satisfactory</td>
</tr>
<tr>
<td>(e.g. 30m, 60m)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sparse</td>
<td>Has residual</td>
<td>Has residual</td>
</tr>
<tr>
<td>(100m)</td>
<td>Worse</td>
<td>Better</td>
</tr>
<tr>
<td>Frequency spectrum</td>
<td>Boosts low frequency energy</td>
<td></td>
</tr>
</tbody>
</table>
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----- Aperture
Numerical Analysis

----- Aperture

Keep the spatial sampling interval of 3m

Reduce aperture gradually: 2400m – 300m – 150m – 75m – 45m

To reduce the edge effect, apply taper at far offset before calculation.
Aperture: 2400m

Red line: \( \text{CdH upgoing wave} \)
Blue line: \( k_x - \omega \) deghosted
Aperture: 2400m

Red line: CdH upgoing wave
Green line: x - ω deghosted
Aperture: 300m

Red line: CdH upgoing wave
Blue line: $k_x - \omega$ deghosted
Aperture: 300m

Red line: CdH upgoing wave
Green line: \( x - \omega \) deghosted
Aperture: 150m

Red line: CdH upgoing wave
Blue line: $k_x - \omega$ deghosted
Aperture: 150m

Red line: CdH upgoing wave
Green line: x - ω deghosted
Aperture: 75m

Red line: CdH upgoing wave
Blue line: $k_x - \omega$ deghosted
Aperture: 75m

Red line: CdH upgoing wave
Green line: $x - \omega$ deghosted
Aperture: 45m

Red line: CdH upgoing wave
Blue line: $k_x - \omega$ deghosted
Aperture: 45m

- Red line: CdH upgoing wave
- Green line: $x - \omega$ deghosted
<table>
<thead>
<tr>
<th>Aperture</th>
<th>$\mathbf{k}_x$ - $\omega$ domain result</th>
<th>$\mathbf{x}$ - $\omega$ domain result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wide (e.g. 2400m)</td>
<td>Ideal</td>
<td>Ideal</td>
</tr>
<tr>
<td>Intermediate (e.g. 300m, 150m)</td>
<td>Has residual</td>
<td>Satisfactory</td>
</tr>
<tr>
<td>Narrow (75m, 45m)</td>
<td>Has residual Worse, artifacts</td>
<td>Has residual Better</td>
</tr>
<tr>
<td>Frequency spectrum</td>
<td>Boosts low frequency energy</td>
<td></td>
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Key Points and Conclusion

• For dense spatial sampling and wide aperture (e.g., in-line data), Green’s theorem de-ghosting techniques in $k_x - \omega$ and $x - \omega$ domains both have ideal results;

• For sparse spatial sampling and narrow aperture (e.g., cross-line data), compared to $k_x - \omega$ domain, the approach in $x - \omega$ domain produces a better result;

• Green’s theorem de-ghosting method in $x - \omega$ domain shows its advantage in boosting low frequencies.
Reference

Thanks~