

A note on P-Vz deghosting derived as a special case of Green's theorem deghosting: the advantages and disadvantages of each method and implications for towed streamer, ocean bottom and on-shore measurements

Arthur B. Weglein, James D. Mayhan,
Lasse Amundsen, and Hong Liang

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The processing flow that are followed before ISS processing

Consider a simple 1D normal incidence example, where in the vicinity of the (towed streamer) cable the pressure field P satisfies:

$$\left[\frac{\partial^2}{\partial z^2} - \frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} \right] P = 0, \quad (1)$$

where c_0 is the wave speed in water, and

$$\left[\frac{d^2}{dz^2} + \frac{\omega^2}{c_0^2} \right] P = 0 \quad (2)$$

is the temporal Fourier transform of equation 1.

The solution of equation 2 is

$$P = \underbrace{A \exp(ikz)}_{\text{down}} + \underbrace{B \exp(-ikz)}_{\text{up}}, \quad (3)$$

where the convention $\exp(-i\omega t)$ is used for going from ω to t . For deghosting, we want to up-down separate P at the assumed measurement location $z = a$. That requires two pieces of information about P .

Two measurements at one depth

If we make the required two pieces of information about P measurements of the field and its derivative at one level, for a cable at $z = a$,

$$P(a) = A \exp(ika) + B \exp(-ika)$$
$$P'(a) = ik[A \exp(ika) - B \exp(-ika)],$$

again with the convention $\exp(-i\omega t)$ is used for going from ω to t .

Two measurements at one depth (continued)

Solve for B ,

$$B = \frac{ikP(a) - P'(a)}{2ik} \exp(ika)$$

and the upgoing wave at $z = a$ is

$$\frac{ikP(a) - P'(a)}{2ik}.$$

Two measurements at one depth (continued)

If we extend the above to a multi-D world in the vicinity of the cable,

$$\left[\nabla^2 - \frac{1}{c_0^2} \partial_t^2 \right] P(x, z, x_s, z_s, t) = 0.$$

In the temporal Fourier domain, this becomes

$$(\nabla^2 + k^2) P(x, z, x_s, z_s, \omega) = 0,$$

and then Fourier transforming over x we have

$$\left[\frac{d^2}{dz^2} + k^2 - k_x^2 \right] P(k_x, z, x_s, z_s, \omega) = 0. \quad (4)$$

Two measurements at one depth (continued)

Equation 4 looks like equation 2 where $q^2 \equiv k^2 - k_x^2$. The solution is

$$P = A \exp(iqz) + B \exp(-iqz),$$

where A, B are functions of k_x and ω , whereas in equation 3, A, B are functions of ω . We get B the same way as before except that the role of k will be played by q , i.e., in the prestack form (equation 4) the deghosted data at the cable (at $z = a$) is

$$P_r(a, k_x, \omega) = \frac{iqP(a, k_x, \omega) - P'(a, k_x, \omega)}{2iq} \quad (5)$$

with $q = +\sqrt{(\omega/c_0)^2 - k_x^2}$.

Two measurements at one depth (continued)

When P' is substituted with $i\omega\rho V_z$ where ρ is the local mass density at the cable and V_z is the vertical component of velocity, equation 5 becomes

$$P_r(a, k_x, \omega) = \frac{iqP(a, k_x, \omega) - i\omega\rho V_z(a, k_x, \omega)}{2iq} \quad (6)$$

the receiver deghosted data on the cable at $z = a$. The latter formula is the proto-type industry standard $P - V_z$ summation for deghosting.

Two measurements at two depths

Another way to provide two pieces of information about P is to use P on the cable and P at the free surface (where $P = 0$). We get

$$P(0) = A + B \quad (7a)$$

$$P(a) = A \exp(ika) + B \exp(-ika). \quad (7b)$$

To solve for B , multiply equation 7a by $\exp(ika)$ and subtract equation 7b to get

Two measurements at two depths (continued)

$$\begin{aligned}\exp(ika)P(0) - P(a) &= B[\exp(ika) - \exp(-ika)] \\ B &= \frac{\exp(ika)P(0) - P(a)}{\exp(ika) - \exp(-ika)} \\ &= \frac{\exp(ika)P(0) - P(a)}{2i \sin(ka)},\end{aligned}\tag{8}$$

which in principle is entirely equivalent to equation 6, but can have stability issues compared to equation 6 for small errors in the cable depth, especially in the vicinity of notches. This was noted in Mayhan and Weglein (2013).

Two measurements at two depths (continued)

To illustrate, let's assume that the total wave is upgoing and it doesn't need deghosting. Then the measured wave is

$$P(z) = P(0) \exp(-ikz).$$

Two measurements at two depths (continued)

Then put $P(a)$ into equation 8 to get

$$B = \frac{\exp(ika)P(0) - \overbrace{P(0)\exp(-ika)}^{P(a)}}{\exp(ika) - \exp(-ika)} = P(0),$$

which is the deghosted data at $z = 0$; if the depth is correct, then the exponentials ($\exp(ika) - \exp(-ika)$) in the numerator and denominator cancel for any frequency and there's no problems. But, if the cable depth is wrong (the cable is at a but you think it's at b), the exponentials don't cancel, and you can get zeros in the denominator.

Two measurements at two depths (continued)

There is no sensitivity in equation 6 to division. Equation 6 is the solution for B with two measurements at one depth, while equation 8 is the same formula for B with two measurements at two depths. In theory equations 6 and 8 are the same, but in practice equation 8 can have issues. Zhang (2007) shows that for small error in depth equation 6 is stable.

Two measurements at two depths (continued)

For typical towed streamer data at 6m, the receiver notch occurs at 125Hz. This frequency is usually outside your data (say max 70Hz). But if you're collecting data to 250Hz, the notch is in your data. The zero is at $ka = \pi$, or $k = \pi/a$. If you make the cable deeper, the notch comes in quicker. At the ocean bottom, the notches can come at 5Hz. Deghosting is very serious for ocean bottom data, because the notches are inside your data. Equation 6 is two measurements (field and its derivative) at one level. That's what Green's theorem depends on, $(P\nabla'G_0 - G_0\nabla'P)$ on the measurement surface.

Derive $P - V_z$ from Green's theorem

Green's theorem-derived deghosting method in 2D is

$$P_r(x, z, x_s, z_s, \omega) = \int dx' \left\{ P(x', z', x_s, z_s, \omega) \frac{\partial}{\partial z'} G_0(x, z, x', z', \omega) - G_0(x, z, x', z', \omega) \frac{\partial}{\partial z'} P(x', z', x_s, z_s, \omega) \right\}, \quad (9)$$

where x', z' are the receiver coordinates, respectively, i.e., x' runs along the cable and z' is the constant depth of the cable, (x_s, z_s) is the source location, and (x, z) is the prediction point, where we choose for deghosting $z_s < z < z'$. The integral equation produces an upwave at x, z , which outputs the receiver deghosted field.

Derive $P - V_z$ from Green's theorem (continued)

The next steps in this derivation benefit from the work of Corrigan et al. (1991), Amundsen (1993) and Weglein and Amundsen (2003). Fourier transforming equation 9 with respect to x gives

$$\begin{aligned} \int \exp(-ik_x x) dx P_r(x, z, x_s, z_s, \omega) &= \int \exp(-ik_x x) dx \int dx' \\ &\times \left\{ P(x', z', x_s, z_s, \omega) \frac{\partial}{\partial z'} G_0(x, z, x', z', \omega) \right. \\ &\left. - G_0(x, z, x', z', \omega) \frac{\partial}{\partial z'} P(x', z', x_s, z_s, \omega) \right\}, \end{aligned} \quad (10)$$

where G_0 satisfies

Derive $P - V_z$ from Green's theorem (continued)

$$(\nabla^2 + k^2)G_0(\vec{r}, \vec{r}', \omega) = \delta(\vec{r} - \vec{r}'). \quad (11)$$

Substitute the bilinear form of the Green's function

$$G_0(\vec{r}, \vec{r}', \omega) = \int \frac{1}{(2\pi)^3} \frac{\exp(-i\vec{k}' \cdot \vec{r}')}{-|\vec{k}'|^2 + k^2 + i\epsilon} \exp(i\vec{k}' \cdot \vec{r}) d\vec{k}', \quad (12)$$

This bilinear form is the plane wave decomposition of G_0 . In 2D,

$$G_0(x, z, x', z', \omega) = \frac{1}{(2\pi)^2} \int \frac{\exp(ik'_x[x - x']) \exp(ik'_z[z - z'])}{-k'^2 + k^2 + i\epsilon} dk'_x dk'_z$$

Derive $P - V_z$ from Green's theorem (continued)

Fourier transform G_0 with $\int \exp(-ik_x x) dx$,

$$\underbrace{\int dx \exp(-ik_x x) \exp(ik'_x x) \exp(-ik'_x x') \exp(ik'_z(z - z'))}_{2\pi\delta(k_x - k'_x)}$$

and the Dirac delta allows you to carry out $\int dk'_x$

$$\exp(-ik_x x') \int \frac{\exp(ik'_z(z - z'))}{-k_x^2 - k_z'^2 + k^2 + i\epsilon} dk'_z. \quad (13)$$

The integral looks like a 1D Green's function if we define $k^2 - k_x^2 \equiv q^2$. The latter relation between q , k_x and k is not due to a dispersion relationship but by introducing and defining the quantity q .

Derive $P - V_z$ from Green's theorem (continued)

The 1D causal solution to

$$\left(\frac{d^2}{dz^2} + k^2 \right) G_0 = \delta$$

is

$$G_0^+ = \frac{\exp(ik|z - z'|)}{2ik}. \quad (14)$$

Derive $P - V_z$ from Green's theorem (continued)

From equation 14 the integral in equation 13 then results in:

$$\frac{\exp(iq|z - z'|)}{2iq},$$

and equation 13 becomes

$$\exp(-ik_x x') \frac{\exp(iq|z - z'|)}{2iq}.$$

Now differentiate equation 13 with respect to z' ,

$$\frac{iq \operatorname{sgn}(z' - z)}{2iq} \exp(iq|z - z'|) \exp(-ik_x x').$$

The other term (in equation 9) will have G_0 with no derivative.

Derive $P - V_z$ from Green's theorem (continued)

Performing the integral over x' we then find

$$\begin{aligned} & P_r(k_x, z, x_s, z_s, \omega) \\ &= P(k_x, z', x_s, z_s, \omega) \frac{\text{sgn}(z' - z)}{2} \exp(iq|z - z'|) \\ &\quad - P'(k_x, z', x_s, z_s, \omega) \frac{\exp(iq|z - z'|)}{2iq}. \end{aligned} \quad (15)$$

It's a combination of P and P' at z' (the measurement depth.) Note there is no sum and no integral. The output point is shallower than the cable, $z' > z$, so $\text{sgn}(z' - z) = 1$ and $|z - z'| = z' - z$.

Derive $P - V_z$ from Green's theorem (continued)

The industry standard practise replaces P' with displacement using the idea sketched here. Start with a 1D Newton's second law:

$$F = ma$$

and in the frequency domain

$$F = mi\omega V_z,$$

where $a = i\omega V_z$ and V_z is the vertical component of velocity.

Derive $P - V_z$ from Green's theorem (continued)

This becomes

$$\frac{F}{A} = \frac{m}{A} i\omega V_z,$$

where A is “area”.

$$P' \sim \frac{1}{l} \frac{F}{A} \sim \frac{\partial}{\partial z} \frac{F}{A} = \frac{m}{Al} i\omega V_z = \rho i\omega V_z, \quad (16)$$

where $\rho = m/(Al)$ is the mass density.

Derive $P - V_z$ from Green's theorem (continued)

When P' is substituted with $i\omega\rho V_z$ where ρ is the local mass density at the cable and V_z is the vertical component of velocity, equation 15 becomes

$$\begin{aligned} & P_r(k_x, z, x_s, z_s, \omega) \\ &= P(k_x, z', x_s, z_s, \omega) \frac{\text{sgn}(z' - z)}{2} \exp(iq|z - z'|) \\ &\quad - i\omega\rho V_z \frac{\exp(iq|z - z'|)}{2iq}. \end{aligned} \tag{17}$$

This is the industry standard and called $P - V_z$ summation.

Derive $P - V_z$ from Green's theorem (continued)

Why are we interested in a Green's theorem solution equation 9 when equations 17 are available?

- 1 In equations 15 and 16 we have to be able to Fourier transform. In the crossline direction, it can be a challenge to perform a Fourier transform because crossline receivers are further apart than inline receivers and crossline aperture is limited compared to inline. Green's theorem allows you to directly input and integrate the data you have recorded. Green's theorem (x, ω) is less upset with missing data, whereas Fourier transforming over x has a more severe requirement (see, e.g., Jing Wu et al., 2013).
- 2 Green's theorem can perform a line or surface integral: on the ocean bottom, or onshore, or for a non horizontal cable. Ghosts are particularly important at the ocean bottom because the notches arrive at lower frequencies and within the seismic bandwidth.

Derive $P - V_z$ from Green's theorem (continued)

We have shown that Green's theorem relates to the industry standard when the measurement surface is horizontal and the data is adequate to perform Fourier transforms. Jing Wu et al. (2013) compared Green's theorem and $P - V_z$ for different numbers of receivers and different distances between receivers. Green's theorem and $P - V_z$ are not the same for a curved boundary. Green's theorem is directly applicable to any shape or form of measurement surface whereas $P - V_z$ is not.

On-shore Green's theorem wave field separation: near surface properties

In the marine application of Green's theorem wavefield separation methods, we assume the source is above the cable and the output point is either above or below the measurement surface. For on-shore application the source can be on (or below) the measurement surface, and we might want the wave separation of the measured data itself. In Mayhan and Weglein (2013) it was shown that using the Green's theorem form that the output point must be more than $1/2 \Delta x$ above the measurement surface, i.e., that $\Delta z \geq 1/2 \Delta x$, where z is the output depth, z' is the cable depth, and Δx is the sampling interval.

On-shore Green's theorem wave field separation (continued)

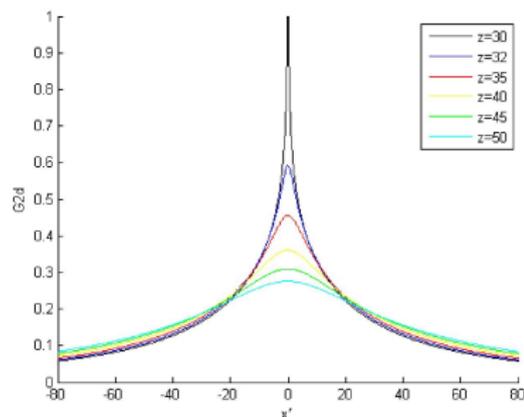
If it gets closer, the calculation becomes unstable, with empirically observed numerical issues. That numerical issue in the x, ω domain (Green's theorem) precludes the output point that is too close to the cable, let alone on the cable. $\Delta z \geq 1/2 \Delta x$ holds for both Green's theorem deghosting and wavefield separation ($P = P_0 + P_s$) in the x, ω domain.

On-shore Green's theorem wave field separation (continued)

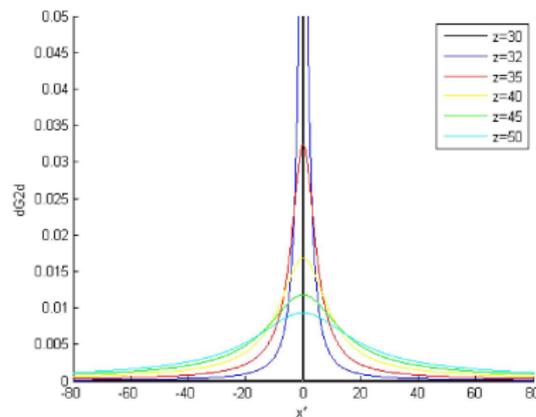
For wavefield separation ($P = P_0 + P_s$),

$$G_0(\vec{r}, \vec{r}', \omega) = -\frac{i}{4} \left(H_0^{(1)}(kR_+) - H_0^{(1)}(kR_-) \right),$$
$$\frac{\partial G_0}{\partial z'}(\vec{r}, \vec{r}', \omega) = -\frac{ik}{4} \left(H_1^{(1)}(kR_+) \frac{z - z'}{R_+} + H_1^{(1)}(kR_-) \frac{z + z'}{R_-} \right),$$
$$R_{\pm} = \sqrt{(x - x')^2 + (y - y')^2 + (z \mp z')^2}.$$

On-shore Green's theorem wave field separation (continued)



$$G_0(x', z'=30, x=0, z, \omega=25)$$



$$\frac{dG_0}{dz}(x', z'=30, x=0, z, \omega=25)$$

For a fixed x , when Δz gets smaller, unless a very small Δx , the value of $\frac{\partial G_0}{\partial z}$ cannot be picked up.

On-shore Green's theorem wave field separation (continued)

From Weglein and Secret (1990) we know that

$$\int_a^b \left\{ P \frac{dG_0^+}{dz'} - G_0^+ \frac{dP}{dz'} \right\} dz' = \begin{cases} -P_s & \text{above } z=a \\ P_0 & \text{below } z=a \end{cases} \quad (18)$$

On-shore Green's theorem wave field separation (continued)

In the example, a is on the measurement surface on the surface of the earth, b is below a , and z_s is above a . The output point is above a or below a . Later in this example, we will make z_s on the surface of the earth, and our output point will be on a , as well. In the world of a whole space of water, the output point above a gives P_s and below a gives P_0 . And in this simple world, separating P_0 and P_s is also deghosting, because it is the same $G_0^+ = G_0^{d+}$. (For deghosting pick $G_0^+ = G_0^{d+}$. In general, deghosting and wavefield separation are not the same.) There is no P_s because there is no up going wave anywhere, including above $z = a$. The source wave is moving down so deghosting gives zero.

On-shore Green's theorem wave field separation (continued)

When we want to compute something where: (1) the source is on the measurement surface, and (2) we want to calculate P_s and P_0 in the data/on the cable. But we can't do that in the \vec{r}, ω domain. Equation 9 makes you stay above the cable (by an amount that depends on sampling), whereas $P - V_z$ has in principle perfect sampling (Δx is zero).

On-shore Green's theorem wave field separation (continued)

For transparency we consider the 1D normal incidence example. In equation 18

$$P = \frac{\exp(ik|z' - z_s|)}{2ik}$$
$$\frac{dP}{dz'} = ik \frac{\exp(ik|z' - z_s|)}{2ik} \operatorname{sgn}(z' - z_s)$$
$$G_0 = \frac{\exp(ik|z - z'|)}{2ik}$$
$$\frac{dG_0}{dz'} = \frac{ik \operatorname{sgn}(z' - z)}{2ik} \exp(ik|z - z'|)$$

On-shore Green's theorem wave field separation (continued)

$$\left|_a^b \left\{ \frac{\exp(ik|z' - z_s|)}{2ik} \left\{ \frac{\operatorname{sgn}(z' - z)}{2} \exp(ik|z - z'|) \right\} \right. \right. \\ \left. \left. - \frac{\exp(ik|z - z'|)}{2ik} \frac{1}{2} \exp(ik|z' - z_s|) \operatorname{sgn}(z' - z_s) \right\} \right.$$

Evaluate at $a < z < b$. a will contribute and b won't contribute. This is shown below.

On-shore Green's theorem wave field separation (continued)

$$\begin{aligned}
 & \frac{\exp(ik(b - z_s))}{2ik} \overbrace{\frac{\text{sgn}(b - z)}{2}}^1 \exp(ik(b - z)) \\
 & - \frac{\exp(ik(b - z))}{2ik} \frac{1}{2} \exp(ik(b - z_s)) \overbrace{\frac{\text{sgn}(b - z_s)}{2}}^1 \\
 & - \left\{ \frac{\exp(ik(a - z_s))}{2ik} \overbrace{\frac{\text{sgn}(a - z)}{2}}^{-1} \exp(ik(z - a)) \right. \\
 & \left. - \frac{\exp(ik(z - a))}{2ik} \frac{1}{2} \exp(ik(a - z_s)) \overbrace{\frac{\text{sgn}(a - z_s)}{2}}^1 \right\}
 \end{aligned}$$

On-shore Green's theorem wave field separation (continued)

$$\begin{aligned} &= \underbrace{\frac{\exp(ik(b - z_s))}{2ik} \frac{1}{2} \exp(ik(b - z)) - \frac{\exp(ik(b - z))}{2ik} \frac{1}{2} \exp(ik(b - z_s))}_{=0} \\ &- \left\{ \frac{\exp(ik(a - z_s))}{2ik} \frac{1}{2} \exp(ik(z - a)) \right. \\ &\quad \left. - \frac{\exp(ik(z - a))}{2ik} \frac{1}{2} \exp(ik(a - z_s)) \right\} \\ &= \frac{1}{2ik} \exp(ik(z - z_s)) = P = P_0 \end{aligned}$$

On-shore Green's theorem wave field separation (continued)

There is no contribution from b . The terms with b 's cancel, and $P = P_0$ because the reference wave is the total wavefield. If we evaluate at $z_s < z < a$, the total contribution is zero because $P_0 = P$ and $P_s = 0$.

On-shore Green's theorem wave field separation (continued)

What to do when you want to put the source on the cable? Fourier transforming into a k_x, ω form avoids the $\Delta z \geq 1/2 \Delta x$ restriction because it begins with $P(k_x, z', x_s, z_s, \omega)$. No integral is left for x . The only question is where do you choose the output point, z ? If you want to deghost on the cable, Fourier transform over x and use the $P - V_z$ forms.

On-shore Green's theorem wave field separation (continued)

The Dirac delta function properties are:

$$\int_V \delta(\vec{r} - \vec{r}') f(\vec{r}') d\vec{r}' = \begin{cases} f(\vec{r}) & \vec{r} \text{ in } V \\ 0 & \vec{r} \text{ outside of } V. \end{cases}$$

The application of Green's theorem methods to either the source or output point on the surface (the measurement surface) boils down to the question of what is $\int_V \delta(\vec{r} - \vec{r}') f(\vec{r}') d\vec{r}'$ when \vec{r} is on the surface enclosing V . You can choose the surface to be in or out of V (Morse and Feshbach, 1953, page 805).

On-shore Green's theorem wave field separation (continued)

In our example above, evaluate at a , when the source is on the cable ($\text{sgn}(z' - z_s) = \text{sgn}(0)$), and if you want the source on the cable to be treated as the source above the cable, then choose $\text{sgn}(a - z_s) = 1$ with $z_s = a$. For the output point, when $z = a$ (predict at the cable), if we want the same sign as when $z > a$, choose $\text{sgn}(a - z) = -1$ when $z = a$. If you want the output point when it is on the surface (measurement surface) to be included with points above the cable choose $\text{sgn}(a - z) = +1$ when $z = a$.

On-shore Green's theorem wave field separation (continued)

So our choice of *sgns* will give P_0 or P_s on the cable, depending on whether you choose the cable to be included with the region below or above the cable, respectively. You're deciding whether the boundary is inside or outside the volume. You can't arrange this in Green's theorem deghosting (x, ω) because you can't get to the boundary, at least not while keeping the algorithm stable.

On-shore Green's theorem wave field separation (continued)

The bottom line here is for land you can't get close enough (to the boundary) to make a decision in Green's theorem deghosting (x, ω) . This is not true if you go to the Fourier k_x, ω domain. But there is no free lunch. If Δx gets too big, $P(k_x, z, \omega)$ becomes inaccurate, and $P - V_z$ can have issues.

Summary

(x, ω) methods for wave separation have advantages compared to (k_x, ω) for limited data (sampling and aperture) and for non horizontal measurement surfaces (ocean bottom, dipping cable). For applications where the interest is in wave separation on the cable itself and where the source is on the measurement surface (on-shore) (k_x, ω) would accommodate that interest whereas (x, ω) (Green's theorem) will not. This paper is examining the implication/differences of a Green's theorem method of deghosting in two domains: (x, ω) and (k_x, ω) . Substituting V_z for P_n , in the (k_x, ω) domain, and benefits/limitations that arise from that substitution (while important) are not within the scope of this paper.

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