Mission - Oriented Seismic Research Program

Inverse Scattering Series and Q: Response of the Internal Multiple Attenuation Algorithm to Absorption and an Application to Bulk Q-estimation

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M-OSRP report pages: 1-11

1. The response of IMAA to an absorptive medium;
2. A potential early stage research for estimating the overburden effect down to a certain reflector without knowing Q;

Inverse Scattering series

1. Free-surface multiples
2. Internal multiple
3. Imaging
4. Inversion

Inverse Scattering series

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Inverse Scattering series

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- Could the output of those task specific procedures provide a bulk estimate of the medium parameters?

Macro view of this talk

1. Q-data modeling
2. IMAA on Q-data
3. Integrated effect of Q down to a reflector
4. A simple exercise and Q-estimation

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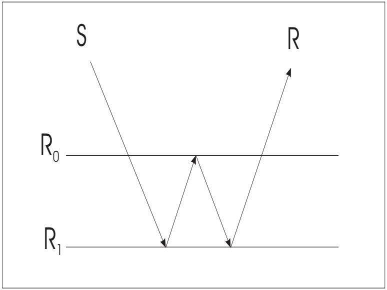

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Macro view of this talk

1. Q-data modeling

- We adopted a constant-Q model, described in [2, 1], where $k_0 = \frac{\omega}{c_0}$ and $k_1 = \frac{\omega}{c_1}\left[1 + \frac{i}{2Q_1} - \frac{1}{\pi Q_1} log\left(\frac{\omega}{\omega_0}\right)\right]$

- We generated the data in ω-domain;

- Inverse Fourier transform the data from ω to t;

Modeling the events

$$
\begin{aligned}
P_1 &= e^{ik_0 z_0} R_0 e^{ik_0 z_0} \\
P_2 &= e^{ik_0 z_0} T_{01} e^{2ik_1(z_1 - z_0)} R_1 T_{10} e^{ik_1 z_0} \\
M_1 &= e^{ik_0 z_0} T_{01} R_1 R_0 R_1 e^{4ik_1(z_1 - z_0)} T_{10} e^{ik_1 z_0}
\end{aligned}
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$T_{01} = 2\frac{k_0}{(k_0+k_1)}$, $T_{10} = 2\frac{k_1}{(k_0+k_1)}$, $R_0 = \frac{k_0-k_1}{k_0+k_1}$ and $R_1 = \frac{k_1-k_2}{k_1+k_2}$.

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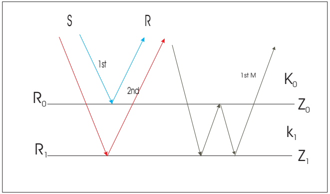

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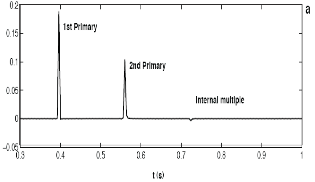

We have begun running the Internal Multiple Attenuation algorithm using data with several values of Q. We make the following initial remarks:

- Internal multiples were more effectively suppressed in cases of larger values of Q;
- Note: At a certain low value of Q (in this case, 20) the amplitude of the internal multiple becomes negligible.

Let us return to the equation of the primary:

$$PR(\omega) = e^{i\frac{\omega}{c_0}z_0} T_{10} e^{i\frac{\omega}{c_1}(z_1 - z_0)} R_1 e^{i\frac{\omega}{c_1}(z_1 - z_0)} T_{01} e^{i\frac{\omega}{c_0}z_0}, \qquad (1)$$

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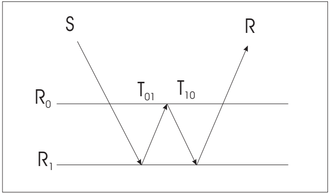
As stated by [3]:

$$PRED = T_{01} T_{10} MULT,$$

where *PRED* is the amplitude prediction of IMAA, and *MULT* is the actual multiple amplitude. Hence we can write:

$$[T_{01} T_{10}] = \frac{PRED}{MULT}$$



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$$PR_Q(\omega) = e^{i\frac{\omega}{c_0}z_0}\mathcal{T}_{10}e^{i\frac{\omega}{c_1}(z_1-z_0)}R_1'e^{i\frac{\omega}{c_1}(z_1-z_0)}\mathcal{T}_{01}e^{i\frac{\omega}{c_0}z_0}, \qquad (2)$$

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and the transmission coefficients can now be written as:

$$\mathcal{T}_{01} = \frac{2c_1[1+F(\omega)/Q_1]^{-1}}{c_0+c_1[1+F(\omega)/Q_1]^{-1}}\exp\left[i\frac{\omega}{c_1}\frac{F(\omega)}{Q_1}(z_1-z_0)\right], \qquad (3)$$

and

$$\mathcal{T}_{10} = \frac{2c_0}{c_0+c_1[1+F(\omega)/Q_1]^{-1}}\exp\left[i\frac{\omega}{c_1}\frac{F(\omega)}{Q_1}(z_1-z_0)\right], \qquad (4)$$

where $F(\omega) = \frac{i}{2} - \frac{1}{\pi}log\left(\omega/\omega_0\right)$

- $$\mathcal{T}_{01}\mathcal{T}_{10}(z_1) = W \exp\left[-\omega \frac{1}{c_1 Q_1}(z_1 - z_0)\right]. \qquad (5)$$

- Let us generalize the last equation to an arbitrary $c(z)$ and $Q(z)$ profile above the reflector at depth z, and let us focus only on the amplitude, hence:

- Let us generalize the last equation to an arbitrary $c(z)$ and $Q(z)$ profile above the reflector at depth z, and let us focus only on the amplitude, hence:
- $$|\mathcal{T}_{01}\mathcal{T}_{10}(z)| \approx \exp\left[-\omega \int_0^z \frac{dz'}{c(z')Q(z')}\right]. \tag{6}$$

According to equation 6, and upon estimating the spectra of the actual and predicted multiple:

$$\left|\frac{\mathrm{PRED}(\omega)}{\mathrm{MULT}(\omega)}\right| \approx \exp\left[-\omega\int_0^z \frac{dz'}{c(z')Q(z')}\right]. \tag{7}$$

If we define QC as the integrated effect of Q down to the reflector:

$$\mathrm{QC} \equiv \int_0^z \frac{dz'}{c(z')Q(z')}, \tag{8}$$

we can write:

$$\text{QC} \approx -\frac{1}{\omega} \log \left| \frac{\text{PRED}(\omega)}{\text{MULT}(\omega)} \right|, \tag{9}$$

and then it is possible to estimate *QC* from any available frequency.

Dealing with some practical issues

As we are dealing with amplitudes, it is very important to be aware of common issues, like possible different multiplication factors:

- Let us assume the following relation with the multiplication factor A:

$$A \left| \frac{\mathrm{PRED}(\omega)}{\mathrm{MULT}(\omega)} \right| \approx e^{-\omega \mathrm{QC}},$$

- A way of getting rid of the unknown factor A, is to calculate the previous equation for a pair of frequencies:

$$\left| \frac{\mathrm{PRED}(\omega_1)\mathrm{MULT}(\omega_2)}{\mathrm{MULT}(\omega_1)\mathrm{PRED}(\omega_2)} \right| \approx e^{-(\omega_1 - \omega_2)\mathrm{QC}},$$

Dealing with some practical issues

Which leads to:

$$\mathrm{QC}(\omega_1, \omega_2) \approx -\frac{1}{\omega_1 - \omega_2} \log \left| \frac{\mathrm{PRED}(\omega_1)\mathrm{MULT}(\omega_2)}{\mathrm{MULT}(\omega_1)\mathrm{PRED}(\omega_2)} \right| . \tag{10}$$

Hence:

- We were able to estimate the overburden effect down to a reflector without any information of the medium..

- We have defined a 1D model, and test it for several different values of Q, according to table 1:

D(m)	c(m/s)	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9	Q_{10}
300	1500	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
480	2200	300	200	150	100	80	75	60	50	30	20
∞	2800	150	100	75	50	40	37.5	30	25	15	10

Table 1. *10 two-layer Earth models. First (left-most) column contains the depths of the layers; second contains the fixed layer velocities; the remaining columns contain the 10 sets of layer Q values used. Q_n is the name of the n'th model.*

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Interval	Freq. 1	Freq. 2
Pair 1	12	16
Pair 2	12	20
Pair 3	20	27
Pair 4	27	47
Pair 5	47	66
Pair 6	66	86
Pair 7	12	86

Table 2. *Frequency pairs tested.*

Model No.	Actual QC	QC-Est.
Q_{10}	0.0040	0.0043
Q_9	0.0029	0.0027
Q_8	0.0016	0.0016
Q_7	0.0014	0.0013
Q_6	0.0011	0.0010
Q_5	0.0010	0.0010
Q_4	0.0008	0.0008
Q_3	0.0005	0.0005
Q_2	0.0004	0.0004
Q_1	0.0003	0.0002

Table 4. *Estimated QC vs. actual QC for all models at freq. pair (12,20 Hz).*

- We expect going in that some pairs of frequencies will be more amenable than others to providing stable, accurate estimation, due to the different effect of Q on higher and lower frequencies.
- The best estimates are *always* found, for all models tested, with the frequency pair (12 Hz, 20 Hz) for each Q-models;

Using equation 11, we are able to back out and provide some bulk estimate of Q.

$$\mathrm{QC}(\omega_1, \omega_2) \approx -\frac{1}{\omega_1 - \omega_2} \log \left| \frac{\mathrm{PRED}(\omega_1)\mathrm{MULT}(\omega_2)}{\mathrm{MULT}(\omega_1)\mathrm{PRED}(\omega_2)} \right| . \tag{11}$$

Model No.	Actual Q	Q-Est.
Q_1	20.00	19.07
Q_2	30.00	28.28
Q_3	50.00	51.25
Q_4	60.00	63.08
Q_5	75.00	82.00
Q_6	80.00	85.11
Q_7	100.00	102.50
Q_8	150.00	169.46
Q_9	200.00	205.00
Q_{10}	300.00	410.00

Final comments

- This work shows an initial stage of a research
- For the simple case presented in this talk, it was possible to use the discrepancy between the IMAA prediction and the actual amplitude of the multiple for providing an estimate of the integrated effect of Q;
- The estimated values provided for our single exercise compared favorably with the actual values
- The answer for that *perennial* question appears to be *yes*!

Acknowledgements

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