

*Society of
Exploration
Geophysicists*

MULTIPLE ATTENUATION

Edited by

*Arthur B. Weglein
William H. Dragoset*

Series Editor

Daniel Ebrom

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Preface

Papers selected for this SEG reprint volume on multiple attenuation sample the geophysical literature from 1948 through 2003. For the past fifty-odd years the presence of multiply reflected energy in seismic data has been a serious issue and challenge for geophysicists. It remains so today. Nevertheless, the seismic exploration community has made dramatic progress in multiple attenuation. This volume chronicles and examines the amazing history and evolution of methods for attenuating multiples. The papers are organized into nine thematic chapters, and appear chronologically (sometimes within subtopics) within each.

The volume begins with a short introduction to some basic concepts of multiple attenuation. Next, Chapter 1 focuses on the era when multiple reflections in seismic data were first clearly seen and characterized. Interestingly, prior to the late 1940's many geophysicists thought that multiples were so weak that they would never be seen at all, much less be a significant problem. Chapter 2 describes some of the first efforts at attenuating multiples – signal processing algorithms that deconvolve or otherwise remove periodic events in seismic traces – as well as some more recent developments using such approaches. In contrast, the papers in Chapter 3 describe methods that attenuate multiples based on moveout discrimination or some other event characteristic that distinguishes them from primary reflections. Papers describing the popular Radon transform-based method of multiple attenuation appear in this chapter.

Chapters 4 and 5 describe two categories of multiple attenuation methods that are directed at accommodating a fully multi-dimensional subsurface: (1) linear methods of modeling and subtracting multiples that require an explicit or implicit model of the reflections that generate multiples; and (2) nonlinear methods that do not require such a model. The latter of these represents a major conceptual advance: the idea that, using nonlinear multi-dimensional wave equation-based methods, multiple reflections can be fully predicted from the information contained within a seismic data set independently of any assumptions or knowledge about the subsurface. Once so predicted, multiples can be subtracted from the original data, yielding a multiple-free result. Because practical applications based on this idea are relatively new, many geophysicists who read this volume may not have been exposed to it during their formal education. Hence, following our Chapter 4 introduction we have included a brief tutorial section that explains the basic concepts of this type of multiple prediction.

There are two complementary ways of thinking about nonlinear wave equation-based multiple prediction. Chapter 4 covers the free-surface and interface model, and Chapter 5 the free-surface and point scatterer model. Compared to earlier methods of multiple attenuation, these multiple prediction schemes place certain conditions on the seismic experiment rather than requiring assumptions about the nature of the subsurface. For example, the source signature must be known, and, as in migration, the data set aperture becomes important. The papers in Chapter 6 discuss some of the consequential difficulties and issues that must be dealt with to make nonlinear wave equation-based multiple prediction practical for field data sets. In particular, many of the Chapter 6 papers describe ways of coping with the 3-D

nature of the primary and multiple wavefields when the acquisition experiment itself is not spatially sampled in a full 3-D sense.

The words “multiple attenuation” immediately make many geophysicists think of marine streamer data. Chapter 7 reminds us that multiples are also an important problem for land data sets and marine data recorded by methods other than horizontally towed streamers. Chapter 8 contains a collection of tutorial papers that emphasize recent multiple attenuation concepts and methodologies. Finally, Chapter 9 presents an alternative approach to dealing with multiple reflections – using them as signal to enhance the subsurface image rather than considering them as noise that must be removed. The serious challenges facing such an endeavor are also described.

Editors of a reprint volume usually face two dilemmas: what criteria are used to select the papers and how many papers should be included. Paper selection was difficult – and necessary – because no reprint volume could contain all the worthy papers on a subject as broad as multiple attenuation. Except for Chapters 1 and 9, we decided to include only papers that directly address the attenuation of multiples. This basic criterion meant that papers about technology that could be used for multiple attenuation, but that contained little or no discussion of multiple attenuation itself, were not selected. Thus, for example, no papers on the fundamentals of the Radon transform or predictive deconvolution appear in this volume. Having established the basic criterion, we then selected papers that we considered to be important and influential. We also polled some of our colleagues and acquaintances for suggestions, especially for early papers about multiple attenuation. Finally, we scanned through the references of selected papers, searching for additional papers that those authors had considered important.

Our objective in this volume is to collect, synthesize, and provide a perspective for the literature on multiple attenuation, an important cornerstone of seismic data processing. Not all of the papers in this volume received a traditional peer review prior to their original publication. In recent years papers in *The Leading Edge*, *First Break*, and the SEG and EAGE *Expanded Abstracts* have formed an increasingly important repository of technical literature. We feel – and we hope that the readers will agree – that including papers from those sources falls within our objective.

Acknowledgments

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Art Weglein and Bill Dragoset
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Volume Introduction

Most of the papers in this reprint volume address the problem of multiple reflections that appear in marine seismic data. In seeking to analyze such data, it helps to classify the recorded events according to the reflections and propagations they have experienced. For typical towed streamer exploration, both the seismic source and the receivers are deployed within the water layer. Figures 1, 2, and 3 show examples of various classes of events for streamer data. The first classification separates events that have experienced the earth's subsurface below the water layer from those that have not. The latter category (Figure 2a) consists of two events:

- the direct arrival, which is energy that travels in a straight-line path from source to receiver, and
- the direct arrival ghost, which is energy that propagates upwards from the source, reflects off the water surface, and then straight to the receiver.

All other recorded events either propagate within the earth or at least encounter the earth at the water bottom. (Seismic energy can reflect from sharp velocity or density contrasts within the water layer. This happens only rarely, so such events are not considered in the classification scheme described here.)

The category of events that experience the earth includes reflections, refractions, diffractions, mode conversions, and an infinite variety of combinations thereof. These events can be further subdivided into two main categories:

- events that propagate downwards from the source and are recorded as up-going waves at a receiver, and
- events that either propagate upwards from the source and/or are recorded as down-going waves at a receiver.

These latter events are called ghosts, of either source or receiver variety, respectively (Figure 2).

Excluding the ghosts, the remaining reflection events can be classified as either primary or multiple depending on the number of upward reflections experienced:

- primary events experience one upward reflection (Figure 1a), and
- multiple events experience two or more upward reflections (Figures 1b-1f).

Here, an upward reflection is defined as one where the incident wave moves away from the measurement surface (that is, the streamer) towards the reflector and the reflected wave moves away from the reflector towards the measurement surface. The incident wave in a downward reflection moves towards the measurement surface and then away from the measurement surface after reflection. Reflections are possible that are neither upward nor downward by these definitions. An example is shown at the right-hand side of Figure 3a. Nevertheless, traditionally, primaries and multiples are defined only in terms of the number and location of upward and downward reflections in their raypaths. The event in Figure 3a has one upward reflection, no downward reflections, and one reflection that is neither; hence, it is a primary.

The final classification defines particular types of multiply reflected events based on the location of their downward reflections:

- A multiple with one or more downward reflections at the free surface (that is, the water surface) is called a free-surface multiple, independent of the rest of its trajectory.
- An internal multiple has all of its downward reflections below the free surface.

Interbed multiple is a common alternative name for internal multiple. Note that although source and receiver ghosts are free surface-related events that experience a downward reflection at the free surface, they do not qualify as free-surface multiples because they were excluded at an earlier step in the classification scheme.

Sound waves in the earth sometimes experience more complicated raypaths than alternating up and down reflections. For example, events that do not easily fit into the classes described above include those that experience refractions or diffractions within their history. The event in Figure 3b has only one upward reflection, but that event is clearly not a primary. The need to accommodate a broader range of events, coupled with advances in multiple attenuation theories and algorithms, suggests the need for more general definitions:

- A prime event (or primary) is a recorded event that cannot be decomposed into other events recorded within the same data set.
- A composite event (or multiple) is a recorded event that can be decomposed into a number of other events that appear within the same data set.

The phrases “can be decomposed” and “cannot be decomposed” refer to whether or not the travel time of the event in question can be expressed as sums and differences of the travel times of other events within the data set (see Figure 3). The concept of “data set” refers to a complete set of shot records that spans sufficient spatial aperture to record all of a multiple’s composite events. Note that the decomposition of higher-order multiples can be accomplished in more than one way, and can include events that are themselves multiples. For example, the multiple in Figure 3f can be decomposed into primary events 1-3, 2-3, 2-4, and 4-5 or alternatively into a multiple event, 1-4, and a primary event, 4-5. This more general scheme of classifying primary and multiple events becomes important in Chapters 4 and 5, which include comprehensive methods that attenuate all events that are composites within the measurement set, including event types beyond the simple upward/downward reflection definition. Further details are presented in the Chapter 4 tutorial.

In the literature, one finds mention of multiple attenuation, elimination, and suppression. Generally, these terms are used interchangeably. In this volume, however, we assign specific definitions to these terms. Attenuation and suppression are synonyms that refer to a process in which the amplitudes of the multiple events in a seismic data set are reduced. Elimination refers to a process in which at least one class of multiples is completely removed from a seismic data set. Thus, elimination is a form of attenuation in which the amplitude reduction is complete. This distinction is important. Some multiple removal algorithms are, at least in principle, eliminators, while others are only attenuators. There is also an important and increasingly significant difference between an explicit prediction of amplitude and phase (time) of a multiple and the assumption that whatever falls on a given traveltimes

trajectory is a multiple to be eliminated. In practice even an elimination algorithm usually accomplishes only partial multiple removal because field data sets seldom meet all of the prerequisites for elimination. This is an important issue, but it is unrelated to the intrinsic capability of the algorithm. The distinction between the intrinsic capabilities of a method and limitations imposed on those capabilities by external factors (such as data collection, subsurface assumptions, etc.) is important for two reasons: an understanding of when application of a procedure is appropriate, and clarity in attributing lack of effectiveness to the proper cause, thereby guiding those seeking better results in the future.

In geophysics, as in all fields of science, progress sometimes appears to be somewhat chaotic. Over a period of many years, however, an evolutionary pattern often emerges from the chaos. Table 1 shows such a pattern for the discipline of multiple attenuation. The leftmost column lists advances in the evolution of multiple attenuation that are presented in Chapters 2 through 5 of this volume. The next column indicates the complexity and realism of the physical models that the attenuation methods can accommodate. More checks indicate higher complexity, realism, and completeness of the physics behind the method. For example, algorithms that attenuate multiples based on periodicity are based on a simple 1-D model of the subsurface, whereas moveout discrimination methods are based on a more complex 2-D ray-tracing model. Together, these two columns represent an overall trend: more complete, realistic physics allows significant advances, and hence improvements, in multiple attenuation. Such improvements reduce the risk of producing poor quality processed seismic data sets, and thus, ultimately, lower the overall risk of hydrocarbon exploration itself. The third column ranks the complexity of the ancillary subsurface information needed by each multiple attenuation method. For example, moveout discrimination requires a nominal velocity profile, and, if one wishes to attenuate selected interbed multiples, the free-surface and interface wavefield method requires specification of a multiple-generating horizon or region. The fourth column describes the need for description of the seismic experiment, such as information about the source signature and the receiver depths. Finally, the rightmost column indicates the data acquisition and computational burden of each method.

Table 1 shows that as the complexity of the physical model that a method can accommodate increases, the burden on data acquisition and processing likewise increases. Simply accommodating a higher dimension of variability in the subsurface requires more thorough surface data acquisition, regardless of the nature of the seismic processing algorithm. Consider the bottom row in the table. Applying a true 3-D version of the free-surface and point scatterer method to attenuate all multiples requires not only a massive computational effort, but also wide-aperture, full-azimuth recorded data. A skeptic, then, might be inclined to ask, "Why develop such an algorithm?" The short answer to this question can be found two rows up the table. When development of the 2-D free-surface and interface model began in earnest during the 1980's, the skeptic, justifiably, could have asked the same question. Yet today, many years later, that technology is being used routinely to process data from large marine streamer 3-D surveys. This example teaches an important lesson for everyone in the exploration geophysics business: *If a method is discovered that solves an important problem, eventually computational technology and data acquisition practices will evolve to*

a point that makes that method practical. This is the nature of progress in the science of seismic exploration.

An interesting, and perhaps counterintuitive, aspect of recent advances in multiple attenuation is that the complexity of the required ancillary subsurface information and, more broadly, the necessary assumptions about the earth's subsurface do not increase as the attenuation algorithms reach their ultimate sophistication. Instead, these new algorithms shift the responsibility for providing detailed subsurface information upwards to the data acquisition at the surface. That shift has an important practical implication – success at removing multiples is no longer limited by uncertainty in subsurface assumptions and information, but instead by the effort one is willing to expend on data acquisition and processing. Prior to these new developments, success in removing multiples was not necessarily commensurate with money spent. With the new technology, however, an Exploration and Production company interested in paying more to achieve better multiple attenuation has that option.

Table 1 could have had many more columns and rows. For example, we could have listed the advantages and disadvantages of each method, their success or failure at attenuating different types of multiples, the years during which the methods were first proposed and then widely practiced, and so on. However, doing that would have spoiled some of the fun readers of this volume will have discovering or rediscovering the history of multiple attenuation.

Multiple attenuation method	Subsurface complexity and reality that method can accommodate	Requirement for ancillary subsurface information and interpretive intervention	Requirement for the description of seismic experiment (e.g., source signature)	Requirements on data acquisition, reconstruction, regularization and computation
Deconvolution based on periodicity of multiples	*	*	*	*
Moveout discrimination methods	**	**	*	**
Free-surface and interface model, 2-D and pseudo 3-D	***	*	***	***
Free-surface and point scatterer model, 2-D and pseudo 3D	****		***	****
Free-surface and interface <i>and</i> free-surface and point scatterer models, true 3-D	*****		***	*****

Table 1: Major stages in the evolution of multiple attenuation.

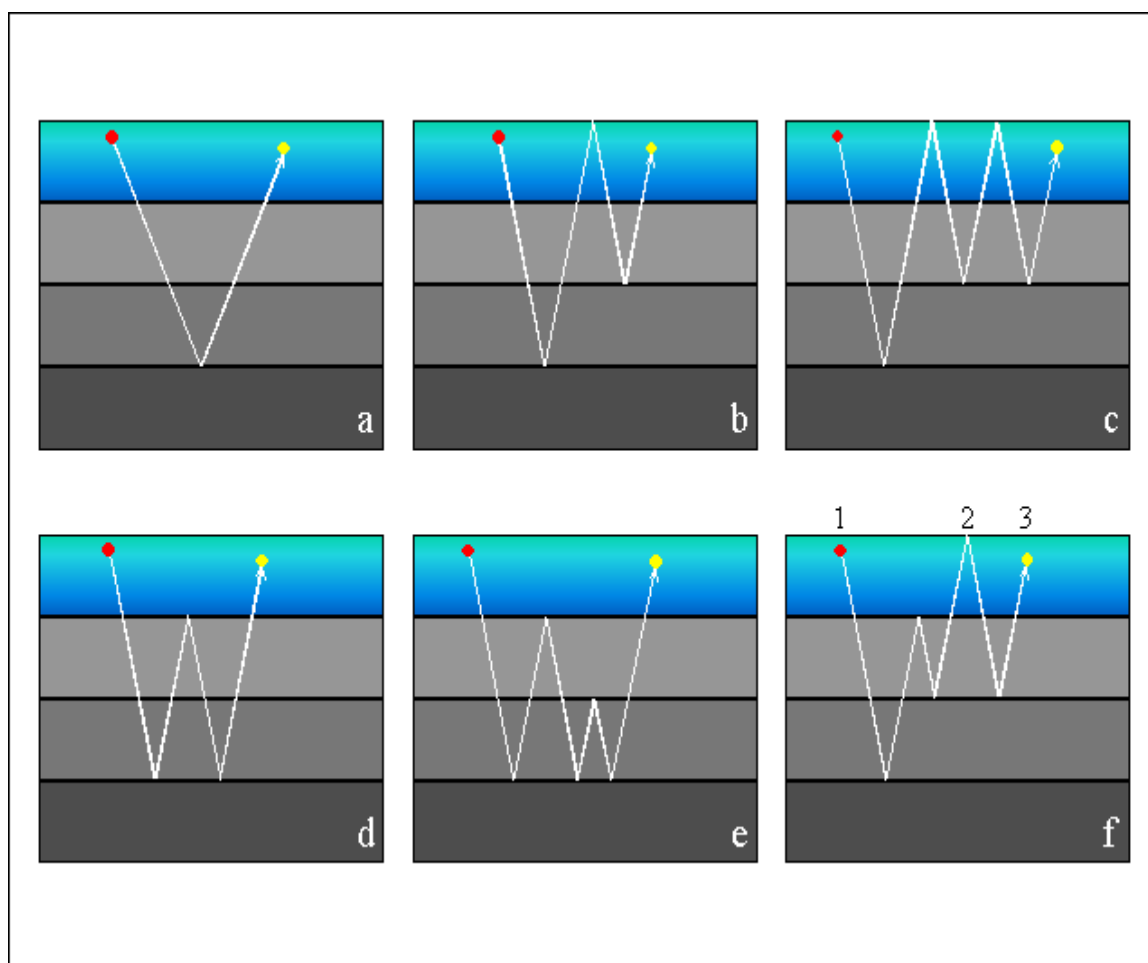


Figure 1: Traditional definition of primary and multiple events. The blue-green area represents the water layer. The red and yellow dots indicate the positions of seismic sources and receivers, respectively. The white lines are raypaths of the events being defined. For the sake of simplicity in the figure, the rays do not refract as they cross reflecting horizons.

a) A primary event has one upward reflection.

b) A multiple event has at least two upward reflections. This example is a 1st-order free-surface multiple because it has a single downward reflection generated by the water surface.

c) A 2nd-order surface multiple.

d) This event is a 1st-order internal multiple because the generating horizon that produces the downward reflection is located in the subsurface.

e) A 2nd-order internal multiple.

f) Hybrid event 1-3 is classified as a free-surface multiple. Even though one generating horizon is below the surface, an algorithm that attacks free-surface multiples will attenuate this event. However, event 1-2, an internal multiple, will remain in the data set after surface multiple attenuation.

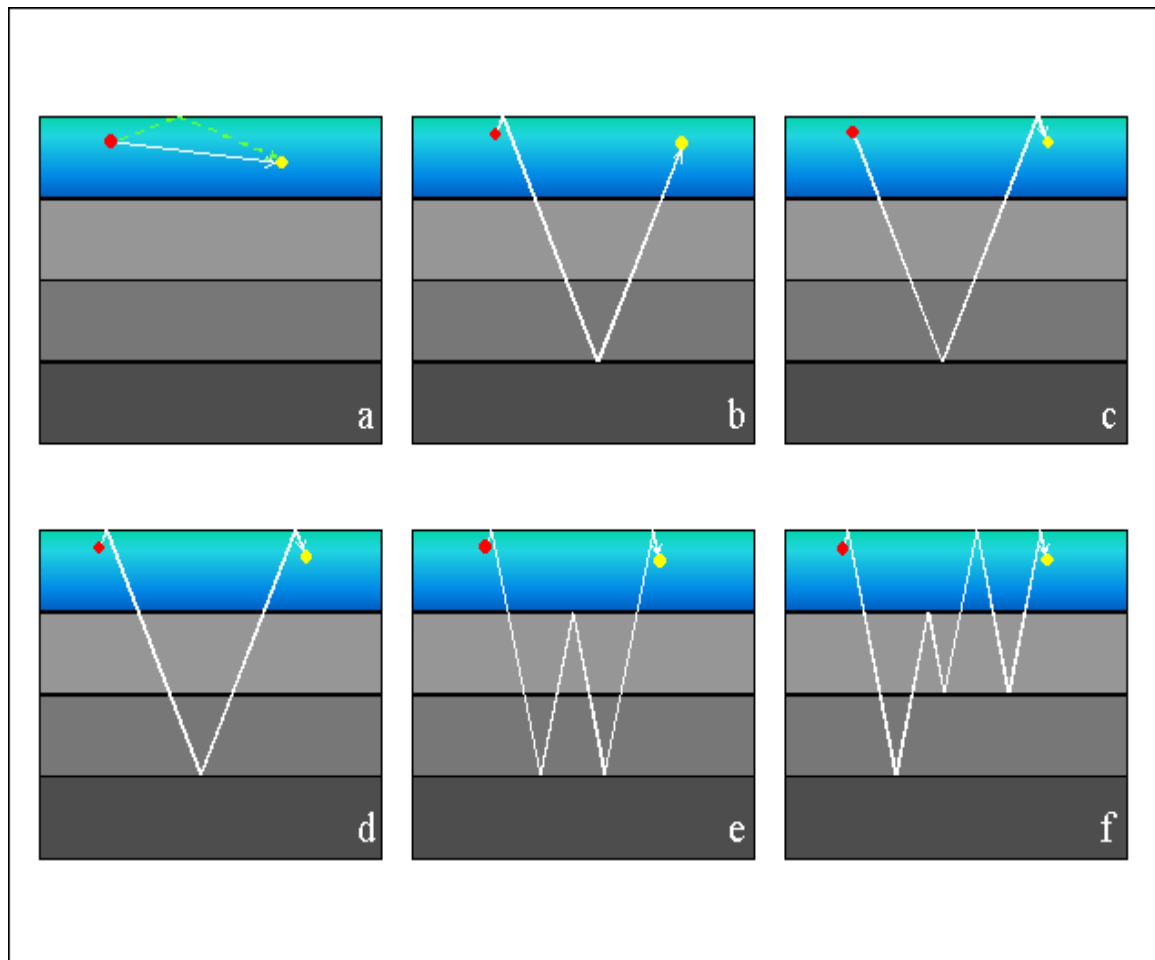


Figure 2: *Definition of direct arrivals and ghosts. The blue-green area represents the water layer. The red and yellow dots indicate the positions of seismic sources and receivers, respectively. The white and green lines are raypaths of the events being defined. For the sake of simplicity in the figure, the rays do not refract as they cross reflecting horizons.*

- a) *A direct arrival (white) and its ghost (green).*
- b) *A source ghost.*
- c) *A receiver ghost.*
- d) *A primary reflection with both a source ghost and a receiver ghost.*
- e) *1st-order surface multiple reflection with both ghosts.*
- f) *2nd-order multiple reflection with both ghosts. Usually, removal of direct arrival and ghost events during seismic data processing is a separate issue from attenuating multiples.*

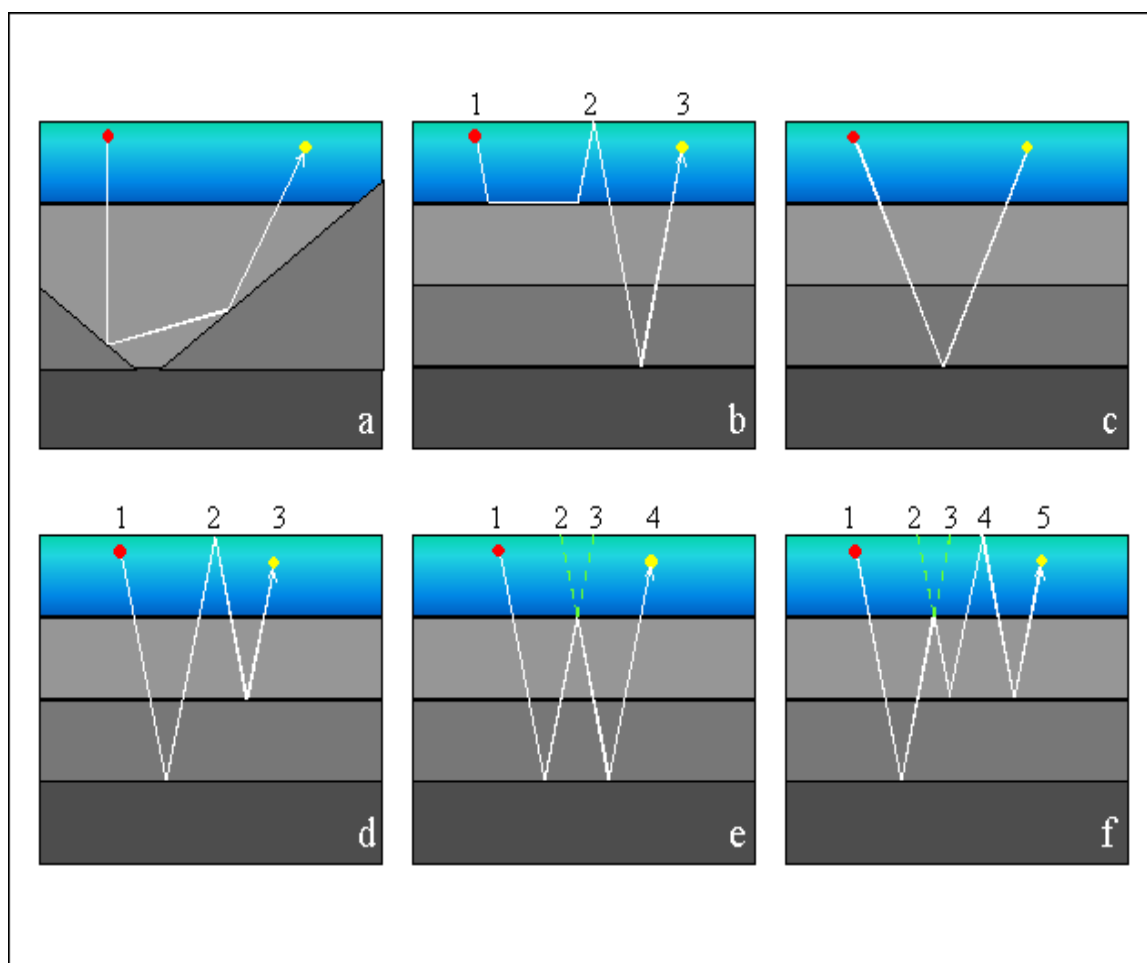


Figure 3: A more general definition of a multiple is: a recorded event that can be decomposed into a number of other recorded events. The dashed green lines in e) and f) represent events whose travel times are subtracted to obtain the multiple's travel time (see below).

a) This event is classified as a primary because it cannot be decomposed into other events.

b) In the traditional classification this event is a primary because it has only one upward reflection. The more general definition classifies it as a multiple since it can be divided into two other recorded events, event 1-2 and event 2-3.

c) This event is a primary reflection. It cannot be decomposed into other recorded events.

d) This event is a multiple because it can be divided into two other recorded events: the primary reflections 1-2 and 2-3. The travel time of the multiple is the sum of the travel times of the two primaries.

e) An internal multiple composed of events 1-3, 2-3, and 2-4. The multiple's travel time is the sum of the travel times of events 1-3 and 2-4, minus the travel time of event 2-3.

f) Another multiple. The sum of travel times 1-3, 2-4, and 4-5, minus travel time 2-3 gives the multiple's travel time.

Chapter 1

Historical papers: characteristics of multiples

From today's perspective the January 1948 issue of *Geophysics* makes for fascinating reading. The topic of that issue was multiple reflections – not so much about how to attenuate them, but rather about whether they even existed in seismograms recorded on land. Consider, for example, the very first sentence in the first paper of this volume: “On several occasions . . . various geophysicists have expressed . . . doubt regarding the existence of multiple reflections . . .” (*Dix, 1948*). Dix went on to argue in favor of the existence of multiples, but in the end he conceded “The evidence . . . does not yet make their existence ‘strictly certain’ . . .” (*Hansen (1948)*), however, had no doubts about the existence of multiples. He reports identifying them by their low average velocities in reflection velocity profiles from Argentina. Interestingly, the translator of Hansen's paper from its original Spanish had to remove the paper's “slightly defensive tone,” which he attributed to the fact that “. . . when Mr. Hansen's paper was originally published . . . few geophysicists and fewer executives reacted favorably to the idea that multiple reflections could be of any practical importance in seismograph exploration . . .”

Many of the papers in the January 1948 issue of *Geophysics* were presented orally two years earlier at a symposium during the SEG's 1946 meeting in Los Angeles. The lead paper of that symposium (*Ellsworth, 1948*) presents solid evidence of multiple reflections in seismic records from Sacramento Valley, California. Ellsworth also describes several kinds of multiples, naming them Type 1, Type 2, and Type 3. Today geophysicists call those types surface multiples, peg-leg multiples, and near-surface multiples. *Johnson (1948)* presents indisputable evidence from Butte County, California of multiple reflections between a basalt layer and the bottom of the weathering zone. Even those geophysicists of the 1948 era who believed that multiples were present apparently were not too concerned. For example, Ellsworth concluded his paper with the statement “. . . the multiple-reflection question as a whole does not seem to present a serious limitation to seismograph interpretation except in isolated cases.” One exception to that widely held viewpoint was Johnson's, who perhaps had a glimmer of the future when he wrote in his paper's conclusion “Thus a highly suspicious attitude toward *every* reflection in areas known to return *some* multiple reflections seems to be justified.”

In marine seismograms geophysicists often observed a mysterious phenomenon dubbed “singing.” As reported by *Werth et al. (1959)* and later by *Levin (1962)*, singing marine seismograms were dominated by sinusoidal, nearly constant frequency energy. Singing was clearly a localized phenomenon, often appearing and then disappearing several times along the length of a single seismic line. Werth et al. describe the recording and analysis of an experiment designed to reveal the cause of singing. They concluded that, at least in their test area, singing was caused by short-period multiple reverberations in the water layer rather than by a wave guide-like excitation of the water layer by the seismic source. Levin's paper describes an experiment to understand the seismic properties of Lake Maracaibo,

which even today has a reputation for producing seismic records that are difficult to process and interpret. Levin found that singing was associated with areas having a high water-bottom reflection coefficient caused by low velocity in gas-saturated bottom mud, a situation certainly conducive to multiple generation.

This set of papers provides a history of significant pioneering and discovery. By their example, they serve to guide and encourage those striving for new scientific understanding.

Chapter 1 Papers

Dix, C.H., 1948, The existence of multiple reflections: Geophysics, **13**, 49-50.

Ellsworth, T.P., 1948, Multiple reflections: Geophysics, **13**, 1-18.

Hansen, R.F., 1948, Multiple reflections of seismic energy: Geophysics, **13**, 58-85.

Johnson, C.H., 1948, Remarks regarding multiple reflections: Geophysics, **13**, 19-26.

Werth, G.C., Liu, D.T. and Trorey, A.W., 1959, Offshore singing – field experiments and theoretical interpretation: Geophysics, **24**, 220-232.

Levin, F.K., 1962, Seismic properties of Lake Maracaibo: Geophysics, **27**, 35-47.

Chapter 2

Multiple attenuation based on a convolutional model

Multiple reflections have many properties that can distinguish them from primaries. The papers in this chapter discuss algorithms that, in essence, attenuate multiples by exploiting one of those properties, their periodicity. Multiply reflected energy is truly periodic only in a 1-D medium. Nevertheless, short-period multiples, such as those that occur in many shallow-water layer areas, are nearly periodic, making simple inverse filtering or deconvolution viable methods of eliminating such multiples. For example, in his classic paper about water reverberations *Backus (1959)* treats the effect of the water layer as an approximate linear filter, and subsequently removes the water-layer reverberations by convolving seismic traces with the inverse of that filter. *Goupillaud (1961)* presents a non-linear generalization of the inverse filter concept, which ideally removes the effect of reverberations in a shallow layer for either land or marine seismic data.

Watson (1965) proposes a simple 1-D scheme that models first-order surface multiples as the convolution of a reflectivity sequence with itself (with an adjustment for the surface reflection coefficient). This leads to an inverse equation for a multiple-free primary trace. Watson approximately solved that equation with a feedback loop procedure. *Anstey and Newman (1966)* discuss the auto-correlogram, which measures the periodicity of seismic traces, and the retro-correlogram (Watson's method under another name), which predicts multiples. They use the auto-correlogram to determine the presence of multiples and suggest, like Watson, the use of a feedback mechanism with the retro-correlogram to attenuate multiples. Many of the papers in Chapter 4 (e.g., *Berkhout and Verschuur, 1997*) discuss a multiple prediction method that is essentially a 2-D generalization of the idea in the Watson and Anstey and Newman papers. The differences are interesting. In Watson's equation (9), for example, wavelet term $c_1(t)$ "approximates the additional filtering provided the multiples by their relatively longer paths in the more highly attenuating near-surface formations." In a 2-D prediction, such a term is not required because the prediction operator accounts for such effects automatically. Anstey and Newman recognized that their 1-D retro-correlogram did not produce accurate results for large offsets or dipping events. A 2-D prediction, on the other hand, does incorporate the effects of offset and dip. The paper by *Kunetz and Fourmann (1968)* offers two efficient schemes for computing 1-D-based multiple deconvolution operators.

The final three papers in this chapter extend the deconvolution approach to multiple attenuation to situations where the multiple reflections are not periodic. *Taner et al. (1995)* accomplish this by multichannel predictive deconvolution performed in the $x-t$ domain. This paper is notable for its thorough introduction to the concept of multichannel deconvolution and how that approach is related to other multiple attenuation methods. *Lamont and Uren (1997)* introduce a "multiple moveout" procedure that makes multiples periodic, and follow that by "isostretch radial trace" to stabilize the wavelet in time. Together, the two transformations pre-condition multiple events to make them suitable for a simple 1-D deconvolution.

Parrish (1998) describes regularizing water-layer multiples spatially and temporarily by migrating the data using the water velocity. This makes single trace dereverberation more effective. After multiple attenuation residual migration relocates the remaining primary reflections to their final positions.

Overall, the papers in this chapter show that the convolutional model has provided tremendous practical value. Simultaneously, it planted the seeds that grew into multi-dimensional wave theoretic methods for dealing with multiple reflections.

Chapter 2 Papers

Backus, M.M., 1959, Water reverberations – their nature and elimination: *Geophysics* **24**, 233-261.

Goupillaud, P.I., 1961, An approach to inverse filtering of near-surface layer effects from seismic records, *Geophysics*, **26**, 754-760.

Watson, R.J., 1965, Decomposition and suppression of multiple reflections: *Geophysics*, **30**, 54-71.

Anstey, N.A. and Newman, P., 1966, The sectional auto-correlogram and the sectional retro-correlogram: *Geophysical Prospecting*, **14**, 389-426.

Kunetz, G. and Fourman, J.M., 1968, Efficient deconvolution of marine seismic records: *Geophysics* **33**, 412-423.

Taner, M.T., O'Doherty, R.F. and Koehler, F., 1995, Long-period multiple suppression by predictive deconvolution in the x-t domain: *Geophysical Prospecting*, **43**, 433-468.

Lamont, M.G. and Uren, N.F., 1997, Multiple elimination using wavefield transformations off the coast of Western Australia: *APPEA Journal*, 777-785.

Parrish, J.F., 1998, Dereverberation after water migration: *SEG Expanded Abstracts*, 1325-1328.

Chapter 3

Multiple attenuation based on event characteristics

Aside from periodicity or near periodicity (see Chapter 2), multiple reflections can differ from primary reflections in other ways that can be exploited to attenuate them. Chief among these is differential moveout. For some seismic data sets simple CMP stacking is effective at reducing the amplitudes of multiples. In the words of Harry Mayne, inventor of the CMP method, “reflections which follow the assumed travel paths are greatly enhanced, and other events are reduced” (*Mayne, 1962*). When stacking alone is insufficient, better separation of primary and multiple events can be achieved by weighted stacking. *Schneider et al. (1965)* demonstrate a method of weighted stacking where the weights are determined by prestack application of optimal filters designed by a multichannel least-squares method. *Schoenberger (1996)* presents an excellent tutorial on the subject of weighted stacking. Although weighted stacking can be quite effective, as Schoenberger demonstrates, it does have a major disadvantage: only stacked traces are output. That limitation can be overcome by 2-D velocity filtering methods. For example, *Ryu (1982)* describes a filter applied to NMO-corrected CMP gathers in the space-time domain that separates multiples from primaries. The velocity function for the NMO lies between the velocity functions for primary events and multiple events. This, in effect, maps the events into different quadrants of the f - k domain, thereby making their separation possible by dip discrimination.

The next seven papers in this chapter describe multiple attenuation based on discrimination in a Radon transform domain. Currently this is the seismic exploration industry’s most popular method of attenuating multiples. In a landmark paper, *Hampson (1986)* reports on multiple discrimination in the parabolic Radon transform domain. An NMO correction was applied to make the originally hyperbolic events in CMP gathers nearly parabolic in the x - t domain, thereby mapping them into approximately discrete points after the parabolic transform. Theoretically, then, multiple discrimination is simple. Provided that a sufficient moveout difference existed originally between the two classes of events, they should map into separate regions in the Radon domain. In practice, unfortunately, things are not so simple. Usually the hoped for discrete points in the parabolic Radon domain are smeared out into overlapping zones of energy, making primary-multiple discrimination difficult. Since Hampson’s 1986 paper, many authors have described refinements to his algorithm. *Yilmaz (1989)* improves on Hampson’s method by replacing the NMO correction prior to the transform with a t^2 -stretching of the CMP data. This converts the hyperbolic events to exact parabolas, improving the velocity resolution and hence the primary-multiple discrimination in the transform domain. Like Yilmaz, *Foster and Mosher (1992)* also set out to improve on Hampson’s method. They describe suppression of multiples using the hyperbolic Radon transform rather than the parabolic transform. Foster and Mosher managed to find an “efficient hyperbolic surface” of integration for the transform stacking that was more accurate than the parabolic surfaces used by Hampson, but without additional computational cost.

In spite of the advances by Yilmaz and Foster and Mosher, Radon transforms were still sometimes a less than satisfactory way of discriminating between primary and multiple events. There were two basic problems:

- The finite spatial sampling and limited aperture in field data records limited resolution in the Radon domain (Thorson and Claerbout, 1985; Sacchi and Ulrych, 1995; Trad et al., 2003).
- In situations with complex geology the apexes of the hyperbolic events might not be at zero offset (as is assumed by a standard Radon transform).

The solution to the first problem was a so-called “high resolution” Radon transform, in which a priori statistical requirements were imposed that forced a sparse distribution of events in the Radon domain. Early forms of this type of transform required an expensive, iteratively re-weighted solution to a least-squares problem (see the references cited above). *Herrmann et al. (2000)* present a relatively inexpensive, non-iterative scheme to solve the problem. Their method is recursive; that is, the weights at each frequency depend on the Radon transform solution found at earlier frequencies. As an alternative, *Moore and Kostov (2002)* suggest a non-iterative, non-recursive scheme that derived the weights from semblance computations along the offset axis. *Hargreaves et al. (2003)* propose a solution to the second problem. Specifically, they address the attenuation of so-called diffracted multiples that arise from scatterers near the ocean bottom. Normal two-dimensional Radon transforms do a poor job of suppressing these multiples because their apexes do not occur at zero offset. Hargreaves et al. show that by adding a third Radon transform parameter, representing the apex location of hyperbolic events, superior separation of primaries and diffracted multiples is possible. *Trad (2003)* shows that a Stolt-type migration operator can be used to implement a very fast apex-shifted Radon transform that performs well in practice.

The design of the reject filter is a problem faced by all 2-D velocity filtering methods, regardless of the domain in which they operate. Ideally, the amplitudes of the transformed multiples and primaries would appear in well-localized and -separated regions so that the data processor could distinguish between these regions. In practice, the amplitudes are often not well-localized and the regions overlap, making selection of the reject filter difficult. *Zhou and Greenhalgh (1996)* solve this problem by using the 2-D transform of wave-equation-based multiple predictions (see Chapter 4) to design the optimal reject filters in the 2-D transform space. Zhou and Greenhalgh have published a series of papers showing that this method can be applied in any 2-D transform domain. Their paper included here discusses application to the parabolic Radon transform domain to attenuate water-layer multiples. *Landa et al. (1999)* extend Zhou and Greenhalgh’s idea to attenuate both surface and interbed multiples.

Multiple reflections can be removed from seismic data by exploiting characteristics other than periodicity or moveout. For example, each multiple in a data set is kinematically and dynamically related to the primary events from the reflecting horizons involved in generating the multiple (see Chapter 4). The two papers by *Doicin and Spitz (1991)* and *Manin and Spitz (1995)* describe a multichannel pattern recognition technique that can target and eliminate a particular multiple based on its relationship with primary reflections. The 1991 and 1995 papers describe 2-D and 3-D applications of this idea, respectively. The method

requires knowledge of the generating mechanism of a targeted multiple.

Various forms of stacking and 2-D velocity filtering generally have the most difficulty separating primaries from multiples at near offsets, where the instantaneous apparent velocity differences between the two kinds of events typically vanish. *Houston (1998)* proposes enhancing the event discrimination at near offsets by applying a localized multichannel coherency filter to gathers NMO corrected with the moveout appropriate for multiples. If multiples are flattened, then they become laterally predictable, whereas overcorrected primary events are not. Houston's coherency filter appears to be more effective than f - k filtering in suppressing multiples without distorting primary reflections. *Hu and White (1998)* describe separating multiples from primaries using another type of coherency-based multichannel filtering called data-adaptive beamforming. This algorithm starts with a beamforming filter based on an initial model of coherent noise (i. e., a multiple) in a data set and then adaptively refines that model to optimize the ability of the beamforming filter to isolate that event. Hu and White show a prestack data example in which the performance of their method was superior to Radon transform-based multiple attenuation.

Finally, *Zhang and Ulrych (2003)* describe a method of separating primaries and multiples based on migration focusing. They first migrate the data prestack, using a velocity function appropriate for multiples, to focus the multiples, but not the primaries. Next, they apply a standard statistical measure, called the median of absolute deviations, along hyperbolic trajectories at the residual velocity of the primaries to identify samples that are outliers. Because only the multiples are focused, such outliers are almost certainly multiples. After replacing these samples with the median and demigrating, the multiples are significantly attenuated.

The papers in this chapter illustrate that attenuation methods based on differences between primaries and multiples are often an effective and appropriate choice within the toolbox of multiple attenuation techniques.

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Chapter 4

Multi-dimensional wavefield methods: Part I

In the volume introduction, we explained that every multiple event in a seismic data set, no matter how complicated, is a composite of two or more simpler events (hereafter called “subevents”) that have their termination points at the free surface. This relationship suggests the possibility of manipulating a data set in a way that uses its subevents to predict its multiples. Indeed, Chapter 2 included several papers that discussed 1-D prediction of multiples from primaries. This chapter and the next include papers that carry this idea beyond 1-D to multi-dimensional wavefield manipulation methods that predict the multiple events in a data set. If such a prediction is sufficiently accurate, multiples can be eliminated simply by subtracting the predictions from the original data. This concept theoretically allows satisfactory multiple attenuation in situations where traditional methods, like those in Chapters 2 and 3, fail. In deep water, for example, simple periodicity-based methods fail because the multiples are not periodic. Furthermore, in recent years the search for petroleum targets has extended to geologic settings beneath complex heterogeneous overburdens such as salt, basalt, karsted sediments and volcanics. The lateral rapid heterogeneity and ill-defined boundaries in these settings are often too complex for any traditional methods of multiple attenuation to accommodate.

Three major methodologies for multiple prediction via multi-dimensional wavefield manipulation have evolved. The *wavefield propagation method* directly models multiples by propagating subevents through one or more cycles of reverberation (e.g., *Wiggins (1988)*). This approach requires a model of the medium, including velocities and reflection coefficients. Thus, it is useful mainly for water layer multiples since the required model information (water velocity and the water-bottom reflection coefficient) is relatively simple. For more complex reverberations, and especially for those in complex geologic settings, direct modeling is not usually considered a viable option because the required a-priori subsurface information is not known sufficiently well. Two alternative approaches, however, provide a capability of predicting multiples that avoid this problem. The *feedback free-surface and interface model* and the *inverse-scattering series free-surface and point scatterer model* are two distinct approaches for predicting both free-surface and internal multiples that reduce or eliminate the need for a-priori subsurface information. This is accomplished by using the data themselves to construct operators that predict the multiples contained within the data (e.g., *Verschuur et al., (1992)*). In particular, both methods can predict surface multiples without any need for a subsurface model. The two methods require knowledge of the acquisition wavelet in order to produce accurate multiple predictions. This chapter includes papers on the wavefield propagation and feedback methods; the inverse series procedures are found in Chapter 5. Because the basic concepts of these latter two methods are not widely known, we introduce them in a brief tutorial immediately following this chapter introduction.

In the first of the wavefield propagation papers, *Bernth and Sonneland (1983)* predict multiples by applying a water layer extrapolation operator in the frequency-wavenumber domain

to prestack data. The predicted multiples are adaptively subtracted from the original data to accommodate timing and amplitude errors in the prediction. *Morley and Claerbout (1983)* use a “Split-Backus” model to predict water layer peg-leg multiples. The modeling assumes near-vertical travel in the water layer, but can handle situations where the source and receiver depths are unequal and the water depth and water-bottom reflection coefficient vary along a line. The multiple modeling procedure suggested by *Berryhill and Kim (1986)* removes two important limitations present in the previous two papers. First, unlike the Bernth and Sonneland algorithm, the method can accommodate any sea-floor profile. Second, unlike the Morley and Claerbout procedure, the prediction is not limited to multiples that propagate nearly vertically in the water layer. The water-bottom reflection coefficient does not appear explicitly in Berryhill and Kim’s multiple prediction procedure; instead, its effects are accounted for in an adaptive subtraction of the predicted multiples from the original data.

Wiggins (1988) derives a method of attenuating water-bottom multiples that allows for a locally varying water-bottom reflection coefficient. The wavefield propagation through the water layer is split into two pieces: a forward-in-time propagation from the surface to the water bottom and a backward-in-time propagation from the surface to the water bottom. The two wavefields at the water bottom then should be identical, trace-by-trace, except for the effects of the water-bottom and water-surface reflectivity. Minimizing the observed difference allows derivation of a filter representing the effects of the water-bottom reflectivity. That filter is then used to calculate the multiple-free wavefield. *Lokshtanov (2000)* describes a wave propagation method that is applied to CMP gathers in the τ - p domain. One advantage of this is that the method easily handles angular dependence of the water-bottom reflection coefficient. Finally, *Hill et al. (2002)* suggest predicting multiples using beam methods to extrapolate the wavefield. A predictive matched filtering to identify and remove multiples from the original data is applied to beam components. Using certain assumptions about the earth model, Hill et al. produce a form of their method that can be used with conventionally recorded 3-D marine data.

As explained in the Chapter 4 Tutorial, the feedback free-surface and interface model is a scheme for the forward modeling of seismic reflection data. When carried out in the seismic data processing, or inverse sense, it provides the opportunity to predict and attenuate multiples that are associated with the reflectors. Early versions of this concept can be found in the landmark works of *Riley and Claerbout (1976)*, and *Kennett (1979)*. Riley and Claerbout begin by using Z -transforms to derive an algorithm that removes surface multiples for a 1-D earth model by, in essence, convolving the data with themselves. Their algorithm includes the inverse of the acquisition wavelet, and they describe how, in some cases, that wavelet can be found using a least-squares minimization of the difference between the seafloor primary convolved with itself and the first-order multiple water-bottom multiple. The paper then addresses 2-D multiple reflections. The authors derive an approximate finite difference-based solution that is analogous to their 1-D solution. The paper concludes with a lengthy discussion of the practical problems expected when the method is applied to less-than-ideal field data sets. Kennett describes construction of a surface multiple suppression operator in the frequency-wavenumber domain for plane-layered elastic and acoustic media.

The fundamental concept is the same as in the Riley and Claerbout paper, but Kennet's method is not restricted to wavefields that travel nearly vertically. Kennett also describes generalization of the method to land data. As is characteristic of the feedback free-surface methods, detailed knowledge of the acquisition wavelet is necessary.

In the early 1980's, Berkhout published a comprehensive treatment of the free-surface and interface model using a feedback formalism (Berkhout, 1982). In particular, Berkhout described an elegant, fully multi-dimensional ω - x formulation that could be described and implemented by simple matrix manipulations. Furthermore, the method placed no restrictions on the nature of the subsurface. This work launched a long-term effort, centered at Delft University, that addressed conceptual and practical issues and eventually brought this method – which is now known as “surface-related multiple elimination” (SRME) – to widespread industry usage. From among the numerous contributions from the Delft group, this chapter includes: *Verschuur et al. (1992)*, *Verschuur and Kabir (1992)*, *Berkhout and Verschuur (1997)*, and *Verschuur and Berkhout (1997)*. Verschuur et al. present the derivation of an ω - x matrix equation for eliminating free-surface multiples. The equation requires no information about the subsurface, but it does require knowledge of the acquisition wavelet. Since that wavelet is typically not well known, the authors propose an adaptive procedure that estimates the wavelet by minimizing the energy in the data after multiples are removed. The matrix equation includes the inverse of a large matrix, which is computationally expensive and may have stability issues. To overcome these problems, Verschuur et al. do a series expansion of the matrix inverse and keep only a few lower-order terms. The authors also mention briefly a recursive scheme for attenuating internal multiples as well (see below). Verschuur and Kabir make a comparison between surface-related multiple elimination and Radon transform multiple elimination. They conclude that the two methods complement each other. This paper also includes a simplified version of the SRME theory, which makes it a good starting point for readers new to the concept. The Berkhout and Verschuur and Verschuur and Berkhout papers are companions: the first is concerned with theory, and the second with practical issues and examples. In these two papers, the authors explain and illustrate an iterative version of the SRME method.

A reciprocity formulation of this free-surface and interface model for multiple removal also had its historical roots and activity centered in Delft University, from the school of Professor A. deHoop. Pioneers like *Fokkema and Van den Berg (1990)* developed these concepts within a wave-theoretical integral equation framework, thereby providing mathematical clarity and physical insight. This approach furthered the understanding of the relationship between the feedback and inverse-scattering methods (see Chapter 5) for free-surface multiple attenuation, and the role, for example, that the obliquity factor (Born and Wolf, 1964) plays in those theories. The wave theoretical angle-dependent obliquity factor is important, particularly for long offsets and shallow targets, as illustrated by three figures from an EAGE Convention paper by *Dragoet (1993)*. The figures, which do not appear in the original published abstract, are included at the end of this chapter introduction.

In addition to those from the Delft group, this chapter includes a few other notable papers on the feedback free-surface and interface model. In the early 1980's Pann filed for a US

Patent, *Pann (1989)*, for a multi-dimensional method of predicting and removing surface multiples that is based on Huygens' principle. Numerically, the steps in Pann's method for predicting multiples of a certain order for a specific trace are the same as those in the Delft scheme. Pann, however, did not reveal any method for easily selecting which combinations of traces need to be convolved. The matrix formulation of the Delft method accomplishes that task automatically. *Dragoset and Jericevic (1998)* derive the equations for SRME in an intuitive fashion by making an analogy between surface multiple prediction and the diffraction aperture problem of classical optics. The Kirchhoff integral solves both problems, and the authors show how the integration can be accomplished by matrix manipulations. They also present and discuss a list of survey design suggestions intended to provide data that are most suitable for the SRME process. Finally, *Al-Bannagi and Verschuur (2003)* propose a method of applying SRME to post-stack data. For a given post-stack trace, the data are demigrated to produce sufficient pre-stack traces for predicting the multiples in the (zero offset) post-stack trace. This approach has several advantages when applying SRME to land data:

- The multiple prediction is performed with traces that have good signal-to-noise ratio compared to that of pre-stack land traces.
- Problems with irregular and sparse spatial sampling of the surface wavefield are avoided.
- Detailed structural variations in the subsurface are accommodated.
- The method is computationally efficient.

As several of the papers in this chapter discuss, the feedback free-surface and interface model method can be applied to the prediction and attenuation of internal multiples. Starting with the free surface and proceeding to the next shallowest reflector, the water bottom, one sequentially predicts and removes first all multiples having downward reflections at the free surface and then all multiples having their shallowest downward reflection at the water bottom. Continuing in this manner, all free-surface and internal multiples are attenuated – one interface at a time. Carrying out this strategy requires accurate depth migration to each interface and good estimation of the reflectivity properties at that imaged reflector. However, extension of the composite event concept to internal multiples allows certain important characteristics of those multiples to be predicted in a simpler fashion. In particular, all internal multiples from a given interface can have their travel times predicted precisely without any knowledge of the subsurface except for the location in time of that reflector. Amplitudes, however, are predicted only approximately with an accuracy that depends on the type of internal multiple. For further discussion of various issues involving internal multiple prediction see the tutorial following this introduction as well as *Coates and Weglein (1996)* and *Weglein and Matson (1998)* in Chapter 5. Here, we include one paper, *Jakubowicz (1998)*, that describes the prediction of internal multiples based on the composite event concept and the free-surface and interface model.

The Delft University feedback formulation for multiple prediction has met with much success, and, in fact, has become an industry-wide standard method of removing multiples. Other related formulations have also led to viable wave equation-based methods of predicting and attenuating multiples. This chapter concludes with one such effort: *Liu et al. (2000)*. These

authors derive multiple attenuation formulas using the invariant embedd

This chapter concludes with one such effort: *Liu et al. (2000)*. These authors derive multiple attenuation formulas using the invariant embedding approach. The results resemble those that appear in the Delft papers. However, the method is implemented in the τ - p domain. For media with only gentle dips, the resulting algorithm is quite efficient because the matrices involved in the multiple prediction are sparse.

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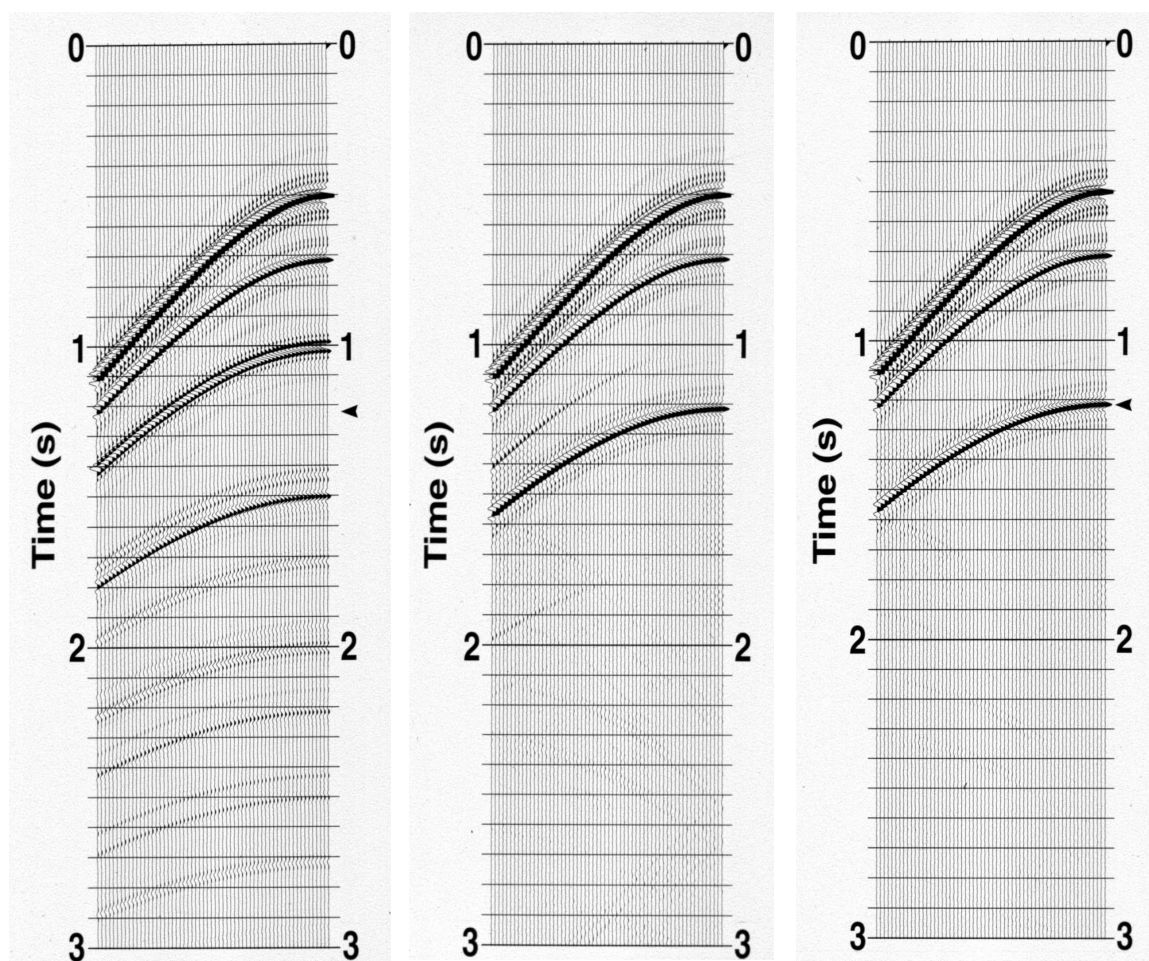


Figure 4: *Figure A: Synthetic marine shot record for testing SRME algorithms. The model has three flat reflecting horizons in a constant velocity medium. The zero-offset arrival times for the three primary events are about 0.5, 0.7, and 1.2 s. The reflection coefficients were chosen such that the third primary event (at the arrow) is exactly cancelled by one of the surface-related multiples. The strong event at 1.0 s and all of the events below 1.2 s are surface-related multiples. Figure B. Surface-related multiples eliminated – no obliquity factor. This result was obtained by directly inverting the matrix at each frequency that represents the SRME operator. That operator is derived from the Kirchhoff integral (Dragoset and Jericevic, 1998). Here, the obliquity factor part of the Kirchhoff integral was ignored by setting it equal to one for all wavefront angles of incidence at the surface. Figure C. Surface-related multiples eliminated – proper obliquity factor applied. Compare this result to that in Figure B. Using the proper obliquity factor improves multiple elimination at the large offsets. This is to be expected, because the raypaths are most oblique to the surface at large offsets.*

Chapter 5

Multi-dimensional wavefield methods: Part II

The free-surface and point scatterer model for the generation of seismic reflection data provides a free-surface model for free-surface multiple generation and a point scatterer model for generating primaries and internal multiples. When this model operates in an inverse sense, the free-surface model removes ghosts and free-surface multiples while the point-scatterer model allows for processing primaries to produce structure maps, earth property estimates, and the removal of internal multiples. The history of this approach derives from a form of perturbation theory called scattering theory and describes how altering (or perturbing) a medium will result in an altered or perturbed wave-field. The tremendous flexibility in scattering theory allows using either a surface, interface or point scatterer model to characterize the difference between the original (reference) and the perturbed medium (actual earth) depending on ones ability to provide or define the difference between reference and earth in an inverse or processing sense. For example, since the air-water boundary is fairly well defined, a free-surface description for that perturbation is chosen and that model is then used to generate and remove free-surface multiples. Since typical subsurface detail is much less well defined, a point scatterer description of the perturbation is chosen for generating and processing primaries and internal multiples.

The origin of the inverse scattering series is found in atomic and nuclear scattering (e.g., Moses, 1956) and was extended to acoustics, (e.g. Prosser (1969), Razavy (1975), and then to seismic exploration by Weglein, Boyse and Anderson (1981). Convergence problems and other practical issues precluded the series, in that pristine form, from providing any practical value. To extract some practical usefulness from this most general and flexible formalism, Weglein, Carvalho, Araujo, and Stolt sought to separate the series into task-specific subseries resulting in distinct algorithms for attenuating free-surface and internal multiples, and to investigate their convergence and practical requirements. *Carvalho et al. (1991, 1992)* developed the free-surface inverse-scattering subseries and then applied it successfully to synthetic data (1991) and to field data (1992). *Araujo et al. (1994)* first identify the subseries that attenuates internal multiples, and exemplify with tests that include free-surface and internal multiples. *Weglein et al. (1997)* present the first comprehensive theory for attenuating all multiples from a multi-dimensional heterogeneous earth with absolutely no information about the subsurface. The excellent convergence properties of these subseries and their ability to accommodate field data were in marked contrast to properties of the overall series.

Coates and Weglein (1996) examine and test the efficacy of prediction of the amplitude and phase of internal multiples for acoustic and elastic media. The phase of all internal multiples is correctly predicted, including that of converted-wave multiples, and the predicted amplitude well attenuates internal multiples of an entire P-wave history. *Weglein and Matson (1998)* use an analytic example to understand the precise nature of the amplitude predicted

in the internal multiple algorithm and provide a sub-event interpretation for the time of the predicted internal multiple phase. *Matson (1996)* provides a map between the forward construction of seismic events in the scattering series and the primaries and multiples in seismic data.

A series describes primaries and multiples in terms of reference propagation and repeated point scatterer interactions where the scattering from any point depends on the difference between actual and reference medium properties at that point. The inverse processes on primaries and multiples require only reference propagation and reflection data. The individual terms in the inverse series that remove internal multiples attenuate all internal multiples of a given order from all interfaces at once – without interpretive intervention or event picking of any kind – and automatically accommodate multiples due to specular, corrugated or diffractive origin. The leading term in the removal series predicts the time of internal multiples precisely; higher-order terms increase the accuracy of the amplitudes of the predicted multiples, thereby allowing better attenuation.

The collaborative project between the feedback group at Delft University and the inverse scattering practitioners at Atlantic Richfield Company helped the latter recognize that providing a range of depths where the troublesome internal multiple at the target might have its downward reflection, allows a truncation of an inner integration in the inverse scattering internal multiple attenuation algorithm, with concomitant significant computational savings. Although the interface method is computational efficient when the reflector causing the downward reflection can be isolated, the ability to automatically attenuate internal multiples under complex geologic conditions without requiring any subsurface information, isolation of reflectors, or interpretive intervention, remains the unique strength of the inverse scattering method of attenuating internal multiples.

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Chapter 6

Multiple prediction for field data sets

Both the multiple prediction method developed at Delft University and by others (Chapter 4) and the method based on scattering theory (Chapter 5) are complete and realistic algorithms from a multidimensional physics point of view. Theoretically, such completeness and realism allow accurate multiple prediction without requiring detailed a priori subsurface information. However, while these methods can accommodate complex wave propagation effects without knowledge of the subsurface, they do place stringent requirements on the definition and completeness of the seismic measurements at the surface. Typically, those requirements are not fully met by present day routine data acquisition practices. This chapter contains papers that describe efforts to overcome the problems that occur when multiples are predicted using less than ideal surface wavefield measurements.

There are two types of stringent requirements. (The Chapter 4 tutorial and *Dragoset and Jericevic (1998)*, also in Chapter 4, discuss the reasons for these requirements.) First, the acquisition wavelet must be either measured or estimated accurately for each shot in the recorded data set. The acquisition wavelet consists of the source signature, the source and receiver array radiation patterns including surface ghost effects, and the recording system filters. Note that the acquisition wavelet is angle dependent and that it does not include any earth filtering effects. Second, the wavefield measured at the surface must be fully sampled and non-aliased. Fully sampled means the following: Suppose a particular recorded trace is defined by its shot and receiver locations at the surface. To predict surface multiples for that trace requires that its shot also be recorded by a 2-D spread of receivers that spans all possible surface locations at which the various multiples in the trace may have their downward reflections. Furthermore, the trace's receiver must record data from a 2-D spread of shots that spans the same surface locations. The size of the aforementioned 2-D spreads – that is, the surface recording aperture – depends on the subsurface structure. For example, if the structure is predominantly 2-D and seismic lines are shot parallel to the dip direction, then 1-D spreads and 2-D multiple prediction suffice. If, however, the subsurface contains 3-D structures – such as ocean-bottom diffractors, reflecting horizons with crossline dip, and salt structures – then 2-D spreads and 3-D multiple prediction are required. Generally, the size of the required crossline aperture is a function of crossline dip.

The requirement for an accurate acquisition wavelet was an important obstacle to the early acceptance and application of wave equation-based multiple prediction technology. An early response to that challenge sought to exploit the multiple attenuation algorithm's sensitivity to having the wavelet by using the algorithm itself to find its own required wavelet. The basic assumption was that seismic data without multiples had fewer events and hence less energy. Therefore, the desired wavelet could be found by searching for the wavelet that produced an energy minimum when multiples predicted using that wavelet were subtracted from the original data. Various incarnations of this concept were introduced. *Verschuur et al. (1992)*

(see Chapter 4) propose a preprocessing deconvolution to remove the angle-dependent components of the acquisition wavelet from the data and follow that with a frequency-dependent energy-minimizing search to determine the residual wavelet amplitude and phase. *Carvalho and Weglein (1994)* describe a global and robust searching of the minimum-energy objective function surface designed to avoid local minima. *Ikelle et al. (1997)* truncate the scattering series formulation of multiple prediction after two terms, which results in a linear relationship between the acquisition wavelet and the free-surface reflections. Ignoring truncation errors, the resulting energy-minimization problem for the wavelet has an analytic solution (that can be refined through an iterative scheme to account for truncation effects). *Dragoet and Jericevic (1998)* (see Chapter 4) formulate the surface multiple attenuation algorithm using a closed-form expression of the multiple prediction series. Unfortunately, that formulation results in the expression for the acquisition wavelet being inside of a rather large matrix that must be inverted, making an iterative wavelet search quite expensive. They show, however, that by applying eigenvalue decomposition to the matrix the acquisition wavelet factor can be isolated in a diagonal matrix, which, of course, is easily and cheaply inverted.

Basically, all of these methods for estimating the acquisition wavelet work at cross-purposes to the underlying physics of the multiple attenuation methods they are meant to serve. For example, destructively interfering events cause no problem for the physics of the algorithms but can cause problems with minimum-energy wavelet estimation approaches. There are other approaches to finding the acquisition wavelet, such as near-field source measurements (*Ziolkowski et al., 1982*), that are independent of the multiple attenuation process. *Ziolkowski, et al. (1999)* propose a wave-theoretical multiple attenuation algorithm that uses near-field source measurements to avoid three problems common to most methods of Chapters 4 and 5. They are: 1) the assumption that the source is a point, 2) the presence of the incident source field in the recorded data, and 3) the need to estimate the wavelet using a minimum-energy criterion. (Note: being a wavefield method, the *Ziolkowski, et al., 1999* paper could have been included in Chapter 4. However, its emphasis on dealing with problems in field data sets makes it at home in this chapter as well.) A variant of Green's theorem known as the extinction theorem provides another possibility for satisfying the need to know the acquisition wavelet (*Weglein et al., 2000*).

Prior to the year 2002, the attenuation methods of Chapters 4 and 5 were typically implemented as 2-D algorithms, and applied to data that were, at best, quasi 3-D. In the presence of 3-D subsurface structures, the resulting timing errors in predicted multiples can be quite large; see, for example, *Ross et al. (1999)*. Furthermore, such timing errors are complicated; they depend on crossline dip, offset, and the order of the multiples. The obvious solution to this problem was to apply 3-D prediction algorithms to 3-D data, but costs generally prohibited this. Therefore, a more pragmatic solution was sought for and developed: sophisticated adaptive subtraction. The general idea behind this approach is that because of imperfections in field data sets and use of prediction algorithms that ignore 3-D complications, effective multiple attenuation can seldom be accomplished in just one step. Instead,

a two-step method is necessary: (1) the multiples are imperfectly predicted followed by (2) adaptive subtraction that compensates for the imperfections, thereby producing reasonably good attenuation in spite of them. Note that adaptive subtraction compensates for prediction errors that arise due to imperfect knowledge of the acquisition wavelet as well as the errors due to use of 2-D rather than 3-D multiple prediction.

Adaptive subtraction can be applied to different data domains (e.g., common shot, common offset, etc.) using many different algorithms, all of which have parameter settings that can affect the results. This flexibility is a mixed blessing: it allows for good results in a wide variety of situations, but the data processing practitioner can face an overwhelming smorgasbord of choices. Using synthetic data, *Abma et al. (2002)* compare the performance of least-squares 1-D matching filters, pattern-matching (Spitz, 1999), and shaped 2-D filters. They conclude that, of those three choices, 1-D matching filters (computed from and applied to a 2-D window of data) are the safest to use for adaptive subtraction. The other two methods, while theoretically more accommodating of errors in the predicted multiples, tend to attenuate primary reflections along with the multiples. *van Borselen et al. (2003)* present a target-oriented adaptive subtraction approach. Their method uses the same 1-D matching algorithm as studied by Abma et al., but it is applied to subtract only multiples having some particular characteristic, such as a predominant dip or frequency range. Presumably, such a constraint minimizes the chance that the adaptive subtraction process will effect primary reflections. *Guo (2003)* describes a more advanced pattern-matching algorithm than that studied by Abma et al. It iteratively calculates the prediction error filter for the primaries and uses a projection signal filter to reduce the effects of random noise. Guo's results on the simple synthetic data sets analyzed by Abma et al. look superior to results of other methods applied to those data sets. In a somewhat different approach, *Ross et al. (1999)* suggest applying time-variant deterministic time corrections to the predicted multiples prior to the adaptive subtraction. The time corrections are derived from a 3-D model of the subsurface.

The need to accommodate errors in multiples produced by applying 2-D prediction to data from a 3-D subsurface was one main impetus behind the development of adaptive subtraction as part of the multiple attenuation process. Although the method of 2-D prediction followed by adaptive subtraction has had many successes, its limitations are also evident. Consider, for example, the offshore Norway data set results displayed by *Hadidi et al. (2002)*. A simple CMP stack (Figure 4) shows a semi-coherent noisy region that is produced by diffracted surface multiples. A 2.5-D application of surface multiple attenuation (see the paper for an explanation of 2.5-D) produced good results when the source-to-receiver crossline separation was small and noticeably poorer results when the source-to-receiver crossline separation was large (Figures 6 and 13, respectively). In neither case was the incoherent noise caused by the diffracted multiples completely attenuated. This result is not a surprise since diffracted multiples are an inherently 3-D phenomenon. The inability to attenuate them fully is due to limitations of current data acquisition practices.

Although possible remedies to the limitations of current data acquisition practices were envisioned at about the same time as when adaptive subtraction became widely used, prac-

tical applications of those ideas have appeared in the literature only recently. Two types of remedies are possible. Although full 3-D marine acquisition with towed streamers may never be practical, there are novel acquisition schemes that offer benefits. Alternatively, data acquired with standard survey designs may be extrapolated and interpolated in various ways and at various processing stages to simulate full 3-D data acquisition. *Keggin et al. (2003)* present a simple, but expensive, acquisition remedy for the problems created by 3-D diffracted multiples. Using an 8-cable streamer ship and a separate shooting ship, a single target swath was acquired nine times with different source-receiver configurations. Stacking the resulting multi-azimuth data sets produced a significant reduction in the noise due to diffracted multiples.

If a 3-D surface multiple prediction algorithm is applied to standard 3-D marine streamer data the sparse crossline sampling of the surface wavefield causes the predicted multiples to be a poor representation of the actual multiples. The next two papers, *van Dedem and Verschuur (2001)* and *Hokstad and Sollie (2003)* attack this problem using sparse inversion (hyperbolic and parabolic, respectively) applied to the crossline multiple contribution gathers. (These gathers consist of the collection of traces that are summed to produce a trace containing predicted multiples.) Specifically, the inversion produces a parametric representation of the information in a sparsely populated crossline multiple contribution gather from which accurate predicted multiples are calculated as if the gather were fully populated. *Nekut (1998)* offers a different solution to the sparse crossline sampling problem of streamer acquisition. He proposes using least-squares migration-demigration as an interpolation and extrapolation process to create a fully populated 3-D data set from standard 3-D field measurements. Although computationally expensive, small-scale synthetic data set tests suggest that the method has promise. The final paper in this chapter, *Kleemeyer et al. (2003)*, is the first exploration industry publication to describe actual application of 3-D surface multiple attenuation to an entire 3-D marine streamer survey. Multiple prediction was accomplished by Shell's MAGIC3D algorithm (*Biersteker, 2001*), which includes a massive data regularization, extrapolation, and interpolation effort. Interestingly, in *Kleemeyer et al.* the predicted multiples are subtracted from the data after both were prestack depth migrated. The results show superior attenuation of diffracted multiples compared to earlier processing efforts.

As this reprint volume goes to press, we think that the seismic industry is just at the beginning of a major effort to develop practical, cost-effective ways of applying 3-D surface multiple prediction to 3-D marine streamer surveys. Although the theory and underlying physical basis of the method are well understood, many pragmatic compromises will likely be necessary and remain to be discovered. While deterministic prediction of multiples brings greater effectiveness, such predictions will never be perfect; hence, there will always be a role for statistical and adaptive procedures. These issues also motivate the drive for more effective data collection, such as single sensor data, and extrapolation and interpolation methods. The heightened demand on definition and completeness will be increasingly satisfied in the coming years, leading to a new level of effectiveness for the attenuation of free surface and internal multiples, and the subsequent imaging and inversion methods for primaries.

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Chapter 7

Multiple attenuation for land, ocean bottom, and vertically-deployed marine receiver data sets

Interestingly, most of the technical literature on multiple attenuation addresses the problem of multiple reflections in marine streamer data. Perhaps, arguing that the relatively high reflection coefficients of the water surface and the water bottom make multiples in marine data more of a problem than those in land data, we should laud geophysicists for tackling the tougher problem. On the other hand, one could counter that their very subtlety makes multiples in land data the tougher problem of the two. Regardless of the outcome of such an argument, it is certainly true that the non-uniformity and noise contamination of typical land data sets makes removing multiples a difficult, expensive problem indeed. Furthermore, because the source of multiples in a land data set is rarely obvious, data processors are often left with a nagging uncertainty: Were the events affected by that process really multiples, or were they perhaps primary reflections?

Kelamis et al. (1990) describe a practical methodology for applying the Radon transform (*Yilmaz (1989)*, Chapter 3) to attenuate multiples in high-density land data sets. By using a pre-processing step to reduce the multiplicity and regularize the geometry, they managed to obtain convincing results at a modest cost. Ten years later, *Kelamis and Verschuur (2000)* investigated the application of surface-related multiple elimination (SRME) (*Verschuur et al. (1992)*, Chapter 4) to land data sets. SRME was originally conceived as a method of eliminating surface multiples in marine data, where the surface has a nearly uniform reflection coefficient. To make this method applicable for land data, Kelamis and Verschuur balanced the data amplitudes in a pre-processing step so as to have smooth multiple prediction operators. They obtained good multiple suppression in land data sets for which other methods (moveout discrimination and predictive deconvolution) were unsatisfactory.

Technologies for recording exploration-quality seismic data on the sea floor have become viable only since about 1990. The main difficulty with sea-floor recording (other than hardware issues) is the impact of the receiver ghost reflection and subsequent water-column reverberations on the data bandwidth. Many years ago, J. E. White (1965) proposed a solution to that problem for pressure measurements. Currently, the seismic industry has many schemes based on White's idea to record both pressure and the vertical component of particle velocity and combine those measurements to cancel reverberations in the water column. *Barr (1997)* presents an overview of this method and *Barr et al. (1997)* compare several different variations of the method. *Amundsen et al. (1998)* describe a generalization of the concept: up/down splitting based on the elastodynamic representation theorem. Unlike earlier methods, their algorithm is valid for a dipping sea floor with medium parameters that vary laterally. *Osen et al. (1999)* extended White's concept to remove water-layer multiples from multicomponent sea-floor data, including the horizontal components of particle velocity. Finally, *Amundsen (2001)* describes an algorithm, based on up/down wavefield

separation, that removes the effect of the free surface entirely as well as accomplishing signature deconvolution. Up/down wavefield separation requires two recordings: either pressure and the vertical component of particle velocity or pressure and its vertical derivative. The method also requires measurement of the direct arrival from the marine source.

Although marine data sets recorded by towed streamers and ocean-bottom sensors differ in many ways, they do have one characteristic in common: the receivers are deployed in a horizontal or near-horizontal plane. *Sonneland et al. (1986)* describe using data recorded with two horizontal marine streamers, one above the other in the vertical plane, to accomplish receiver deghosting. Their algorithm is based on up/down wavefield separation, and thus, as in *Amundsen (2001)* allows both dereverberation and designature. An alternative vertical acquisition scheme is to deploy entire marine cables vertically, a method developed during the 1990's. As with the borehole VSP method, vertical marine cables allow separation of the recorded pressure wavefield into its up- and down-going components. That capability by itself does not, however, solve the multiple problem. *Wang et al. (2000)* describe attenuating multiples in vertical cable data using a three-step process: a common-shot tau-p filter to remove receiver ghost multiples, common-receiver deconvolution to remove source ghosts, and a Radon filter to remove other multiples.

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Chapter 8

Tutorials, reviews and case histories

This chapter represents a collection of papers that provide: (1) a perspective of the broad landscape of techniques and approaches to attenuating multiples, e.g., papers (26,66), (2) a detailed description of a particular method, plus an attempt to bridge to another approach with similar objectives but with a different language and history, seeking to better understand and differentiate between substantive (rather than cosmetic) differences and strengths/weaknesses, e.g., papers (60,43,67,7 and 68) and (3) case studies that exemplify different techniques with field data comparisons and where relative effectiveness is described and plausibility arguments proposed to explain these differences, e.g., papers(44,75).

Basically, there has been a rejuvenated interest in multiple attenuation over the past few years- due to the industry trend to deep water, with a collection of technical and economic challenges driving the need for reduced risk and increased reliability. A direct response to this challenge has come from new thinking, testing, development and application on multi-dimensional wave theoretic multiple removal techniques that do not require knowledge of the subsurface. This moves the boundary between deterministic and signal processing/statistical methods into the region formally occupied by the latter. There are, and always will be, aspects of the recorded reflection data that are outside any chosen deterministic model- and progress and effectiveness in attenuating multiples on field data is determined by how well we can address that critical component, as well. Sometimes those non-deterministic approaches are explicit, with sophisticated statistical models, e.g., to model the difference between signal and random noise. Other times the procedures are implicit and subtler with a call for adaptive and data example specific parameter estimation: a recognition that the chosen deterministic model needs some user intervention and assistance, to accommodate model inadequacy or prerequisite satisfaction.

As a general guideline and guiding principle to choosing multiple removal algorithms, a tool box approach seems appropriate -where the approach is chosen by a set of factors including: (1) geologic complexity of multiple generating reflectors: horizontal, deep, specular, corrugated and diffractive, (2)cost/benefit, (3) seismic processing objective, and (4) availability of data, algorithm and satisfaction of practical prerequisites.

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Chapter 9

Multiples as signal

There are two basic views of seismic reflection data. The inclusive view treats all reflection data as signal, since all events contain information about the subsurface. In contrast, the exclusive view treats only primary reflections as signal and considers multiples an undesirable noise. The latter view – that espoused by the first eight chapters of this volume – by far dominates, for a good reason. Extracting accurate depth images from just primary reflection information is, in general, a complex, difficult task. Conventional wisdom is that imaging with multiples must be significantly more challenging. As the papers in this chapter illustrate, however, challenging conventional wisdom can lead to surprising and promising ideas. Specifically, extracting useful information from multiple reflections may not be as difficult as traditionally thought.

Reiter et al. (1991) demonstrate the use of receiver-side, first-order surface multiples to enhance migrated primary reflection images in data recorded on the ocean bottom. This was accomplished by using moveout discrimination to isolate the multiples of interest and then migrating them using a ray-based Kirchhoff depth migration. The resulting composite image had improved signal-to-noise ratio and extended lateral subsurface coverage. In contrast to Reiter et al., *Berkhout and Verschuur (1994)* propose a general scheme for migrating all surface multiples in a data set rather than just those with a specific ray path. The method requires isolation of the surface multiple component of the seismic response and the formation of areal source wavefields using the total seismic response. The authors note that the method can be extended to internal multiples, although no details are given. *Sheng (2001)* suggests yet another method of using the information in surface multiples. First-order surface multiples are cross-correlated with primaries to remove the first leg of their raypath. The remaining second leg is migrated using the appropriate primary migration operator. *Youn and Zhou (2001)* describe a migration scheme that migrates a source function in the forward direction and the recorded traces in a shot record in the backward direction, using – for both cases – a full two-way scalar wave equation. The two propagated wavefields are correlated and summed over all time indices to produce an image frame. The claim is that this scheme uses all types of events to produce a depth migrated image directly from raw field records. Because of the massive data storage and computational requirements, the authors applied the algorithm to only an extremely decimated version of the synthetic Marmousi model. Finally, *Berkhout and Verschuur (2003)* introduce the notion of a “focal transform,” a variation on the feedback model method of multiple prediction (see Chapter 4) that involves correlation rather than convolution. The focal transform reduces the order of each surface multiple in a data set by one. This allows primary information to be extracted from multiples. One application is the extracting of short-offset primary reflections (that are not directly recorded) from longer-offset multiples (that are directly recorded).

The papers in this chapter clearly support the premise that the multiple wavefield in a seismic data set contains useful information about the earth’s subsurface. Thus, arguably a better

approach to the multiple problem is using that information rather than discarding it. Based on the papers included here, data correlation seems to be a key to making use of multiples. However, use of multiply reflected energy is still a relatively unexplored terrain. Other means of exploiting, rather than attenuating, multiple energy may remain to be discovered. We expect further efforts in this area in the years ahead.

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