Notes

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1 Directly computing the wavelet, radiation pattern and deghosted data from a single shot record

The wavefield P can be computed above the cable from P along the cable (H. Tan 1999).

$$P(x,z,x_s,z_s;\omega)=\int P(x',z_c,x_s,z_s,\omega)\left[\frac{\partial G_0^{DD}(x,z,x',z',\omega)}{\partial z'}\right]_{z'=z_c}dx' \quad (1)$$

where $z_c = z_{cable}$.

The vertical derivative of the measured wavefield can be computed by taking $\frac{\partial}{\partial z}$ of (1) and evaluating at $z = z_c - \epsilon$ (slightly above the cable: $+z$ points downward):

$$\lim_{\epsilon\to 0^+}\left(\frac{\partial P}{\partial z}(x,z,x_s,z_s;\omega\right)_{z=z_c-\epsilon} \quad (2)$$
$$=\lim_{\epsilon\to 0^+}\int P(x',z_c,x_s,z_s;\omega)\left[\frac{\partial^2 G_0^{DD}(x,z,x',z',\omega)}{\partial z\partial z'}\right]_{\substack{z'=z_c\\ z=z_c-\epsilon}}dx'.$$

The extinction theorem allows an integral that involves P and $\frac{\partial P}{\partial z}$ along the cable to compute P_0 for $z > z_c$ and $-P_s$ for $z < z_c$ (see Weglein and Secrest 1990):

$$\left\{\begin{array}{ll} P_0(x,z,x_s,z_s;\omega) & z<z_c \\ P_s(x,z,x_s,z_s;\omega) & z>z_c \end{array}\right\} = -\int\{P(x',z_c,x_s,z_s;\omega)\left[\frac{\partial G_0^D(x,z,x',z',\omega)}{\partial z'}\right]_{z'=z_c} \quad (3)$$
$$-\left[\frac{\partial P(x',z',x_s,z_s;\omega)}{\partial z'}\right]_{z'=z_c}G_0^D(x,z,x',z_c,\omega)\}dx'$$

Evaluate (2) at

$$(x,z,x_s,z_s)\longrightarrow\lim_{\epsilon\to 0^+}(x,z,x_s,z_s)_{z=z_c-\epsilon} \quad (4)$$

to find

$$\lim_{\epsilon\to 0^+}\left(\frac{\partial P(x',z',x_s,z_s,\omega)}{\partial z'}\right)_{z'=z_c-\epsilon} \tag{5}$$
$$= \lim_{\epsilon\to 0^+}\int P(x'',z_c,x_s,z_s;\omega)\left[\frac{\partial^2 G_0^{DD}(x',z',x'',z'',\omega)}{\partial z'\partial z''}\right]_{\substack{z'=z_c-\epsilon\\ z''=z_c}} dx''$$

and substituting (5) into (3), the right hand member of equation (3) becomes

$$\int\left\{P(x',z_c,x_s,z_s;\omega)\left[\frac{\partial G_0^D(x,z,x',z_c,\omega)}{\partial z'}\right]_{z'=z_c}\right\}dx' \tag{6}$$
$$-\lim_{\epsilon\to 0^+}\{\int P(x'',z_c,x_s,z_s;\omega)$$
$$\frac{\partial^2 G_0^{DD}(x',z',x'',z'',\omega)}{\partial z'\partial z''}G_0^D(x,z,x',z',\omega)dx''dx'\}_{\substack{z'=z_c-\epsilon\\ z''=z_c}}.$$

We can eliminate the need for $\frac{\partial P}{\partial z'}$ by using equation (2) and the fact that P and $\frac{\partial P}{\partial z'}$ are continuous functions in space so that the limit from above the cable can be used for the value on the cable

$$\lim_{\epsilon\to 0^+}\left[\frac{\partial P(\overrightarrow{r}',\overrightarrow{r_s};\omega)}{\partial z'}\right]_{z'=z_c-\epsilon} = \frac{\partial P(x',z_c,\overrightarrow{r_s};\omega)}{\partial z'}. \tag{7}$$

We have

$$\left\{\begin{array}{llll} +P_0(x,z,x_s,z_s;\omega) & for & z & below \quad z_c \\ -P_s(x,z,x_s,z_s;\omega) & for & z & above \quad z_c \end{array}\right\}$$
$$= -\int dx_g P(x_g,z_c,x_s,z_s;\omega)\{\left[\frac{\partial G_0^D(x,z,x_g,z';\omega)}{\partial z'}\right]_{z'=z_c}$$
$$-\lim_{\epsilon\to 0^+}\int\left[\left(\frac{\partial^2 G_0^{DD}(x',z',x_g,z'',\omega)}{\partial z'\partial z''}\right)_{\substack{z'=z_c-\epsilon\\ z''=z_c}} G_0^D(x,z,x',z_c,\omega)\right]dx'\} \tag{8}$$

The G_0, G_0^D, and G_0^{DD} are all causal in these extinction theorem applications.

2 Receiver deghosted data from the field along the cable

Define

$$W(x,z,x_g,z_c,x_s,z_s;\omega) \equiv \left[\frac{\partial G_0^D(x,z,x_g,z';\omega)}{\partial z'}\right]_{z'=z_c-\epsilon} \tag{9}$$
$$-\int\left[\frac{\partial^2 G_0^{DD}(x',z',x_g,z'',\omega)}{\partial z'\partial z''}\right]_{\substack{z'=z_c-\epsilon\\ z''=z_c}} G_0^D(x,z,x',z_c;\omega)dx'$$

[G_0^D and G_0^{DD} are analytic functions (see Morse and Feshbach, Chapter 7)].

RESULT: A single weighted integral over x_g of the data on the cable for a single shot record,

$$\int P(x_g, z_c, x_s, z_s; \omega) W(x, z, x_g, z_c, x_s, z_s; \omega) dx_g \tag{10}$$

produces (for that shot record) the reference wavefield, P_0, (wavelet and radiation pattern) for all (x, z) with $z > z_c$ (i.e. at any point below the cable) and the scattered field (actually $-P_s$) for all (x, z) with $z < z_c$ (i.e. at any point above the cable). When the reference field, P_0, is due to a localized source, then $P_0 = A(\omega) G_0^+(\overrightarrow{r}, \overrightarrow{r_s}; \omega)$ where $A(\omega)$ is the source signature (phase and amplitude) and *is directly computable* from the data recorded with the cable for each shot record.

SIGNIFICANCE: The total measured wavefield, P, integrated over the cable produces the amplitude and phase of the wavelet (and source radiation pattern) for that shot record without a second cable, or well, or 1-D, or statistical assumptions.

There is a rejuvenated interest in estimating the source signature (and pattern) for free surface and internal multiple attenuation. Current methods for estimating the wavelet (for those applications) can keep the underlying power of the demultiple techniques (e.g., for separating interfering primary and multiple events) from reaching their full potential. This wavelet estimation method makes no assumption that is at cross purposes with the underlying capability of the inverse scattering demultiple methods.

Deghosting: We start by observing that for each shot-record, the receiver deghosted data can be computed directly from the measured total field, P, and its normal derivative $\frac{dP}{dz'}$ along the cable.

$$\begin{aligned} P^{deghosted}(\overrightarrow{r}, \overrightarrow{r_s}; \omega) &= \int_{z'=z_c} \{P(\overrightarrow{r}', \overrightarrow{r_s}; \omega) \frac{\partial G_0^+(\overrightarrow{r}, \overrightarrow{r}'; \omega)}{\partial z'} \\ &\quad - G_0^+(\overrightarrow{r}, \overrightarrow{r}'; \omega) \frac{\partial P(\overrightarrow{r}', \overrightarrow{r_s}; \omega)}{\partial z'}\} dx' \end{aligned} \tag{11}$$

where G_0^+ is the whole-space causal Green's function (in 3-D) $G_0^+ = -\frac{1}{4\pi} \frac{e^{ik|\overrightarrow{r} - \overrightarrow{r}'|}}{|\overrightarrow{r} - \overrightarrow{r}'|}$ and the evaluation point $\overrightarrow{r}$ is above the cable, i.e. $z > z_c$. We have

$$\begin{aligned} P^{deghosted} &= \int \{P(x', z_c, x_s, z_s; \omega) \left[\frac{\partial G_0(x, z, x', z'; \omega)}{\partial z'} \right]_{z'=z_c} \\ &\quad - G_0(x, z, x', z_c; \omega) \lim_{\epsilon \to 0^+} \int P(x'', z_c, x_s, z_s; \omega) \left[\frac{\partial^2 G_0^{DD}(x', z, x'', z''; \omega)}{\partial z' \partial z''} \right]_{\substack{z'=z_c-\epsilon \\ z''=z_c}} dx'' \} dx'. \end{aligned} \tag{12}$$

Substituting $\frac{\partial P}{\partial z'}$ from equation (2) in equation (13) we find

$$\begin{aligned} P^{deghosted}(x,z,x_s,z_s;\omega) &= \int dx_g P(x_g,z_c,x_s,z_s;\omega)\{\left[\frac{\partial G_0(x,z,x_g,z';\omega)}{\partial z'}\right]_{z'=z_c} \\ &- \lim_{\epsilon\to 0^+}\int[\left(\frac{\partial^2 G_0^{DD}(x',z',x_g,z'',\omega)}{\partial z'\partial z''}\right)_{\substack{z'=z_c-\epsilon \\ z''=z_c}} \\ &G_0(x,z,x',z_c;\omega)]dx'\}. \end{aligned} \quad (13)$$

Equation (14) produces deghosted P for all points above the cable.

3 Summary

The new results of this section derive from an evolution of ideas, Weglein and Secrest (1990), H. Tan (1992), Osen et al (1998), H. Tan (1999), Weglein, Tan et al. (2000) that provide the opportunity to estimate the wavelet and deghost your data from an integral over your shot record without: sensitive division operations, the need for either an extra towed streamer or well information, or any finite difference or Taylor series approximations.

The wavelet, $A(\omega)$, for each shot record is

$$A(\omega) = \frac{\int P(x_g,z_c,x_s,z_s;\omega)W_1(x,z,x_g,z_c,x_s,z_s;\omega)dx_g}{G_0(x,z,x_s,z_s;\omega)} \quad (14)$$

for any (x,z) below the cable. This freedom to choose the evaluation point (x,z) provides (in practice) a plethora of estimates for $A(\omega)$. Delima et al. (1990) exploited this freedom to provide robust estimates of $A(\omega)$ when P and P_n were measured. Here equation (15) provides a similar suit of estimates for $A(\omega)$ with only P on the cable.

$$P^{deghosted}(x,z,x_s,z_s;\omega) = \int P(x_g,z_c,x_s,z_s;\omega)W_2(x,z,x_g,z_c,x_s,z_s;\omega)dx_g \quad (15)$$

for any (x,z) above the cable.

W_1 and W_2 are given by equation (7) or what equation (7) becomes when G_0^D is replaced by G_0, namely equation 14, for wavelet or deghosting application. All of these extinction theorem applications require single sensor data, i.e. a measurement of P that retains both P_s (reflection data) and P_0 the reference wavefield.